CIRCUITS (1)

solved exams & examples.

Contents:

2.5 L.E.

First

Exams (2008 final+midterm2010/2011)

Alexandria University
Faculty of Engineering
Electrical Eng. Dept.
Jan.2008

Course titled: Number: Electrical Circuits (EE x11)
First Year

Time allowed: Three Hours

Answer all the following six questions:

1- Use the principle of superposition to find i_1 and i_2 in the circuit shown in fig.(1).
Then, determine which sources are generating power.

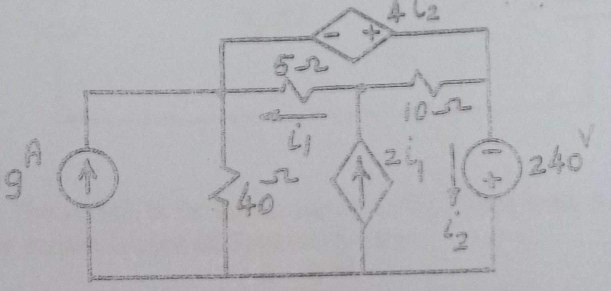


Fig.(1)

(15 marks)

2- For the circuit shown in fig.(2):

a) find the value of R that enables the circuit to deliver maximum power to the terminals a and b. b) Find the maximum power delivered to R.

c) When R is adjusted for maximum power transfer, how much power is the 100 V source delivering to the circuit ?

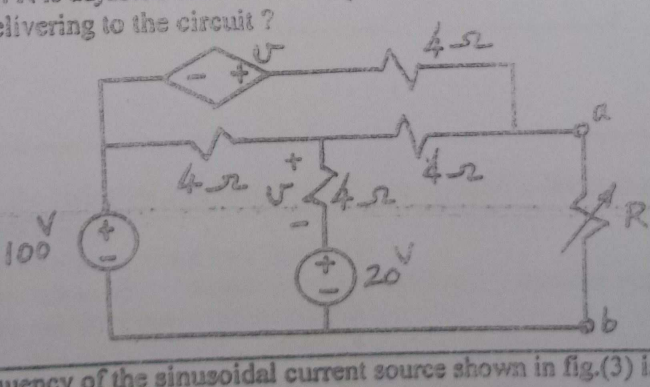


Fig.(2)

(15 marks)

3- The frequency of the sinusoidal current source shown in fig.(3) is adjusted for unity power factor resonance. Calculate:

a) The resonant frequency ω_0. b) The quality factor of the circuit.

c) The amplitude of $v(t)$ when $i_s(t) = 50 \sin \omega_0 t$ mA.

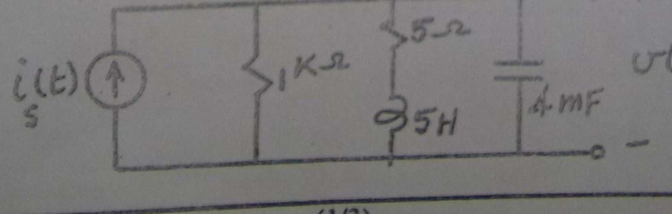


Fig.(3)

(15 marks)

4- a) The switch in the circuit shown in fig.(4) has been closed for a long time before opening at $t = 0$. Find:

i) $v_o(t)$ for $t \geq 0$.

ii) The initial energy stored in capacitors at $t = 0$.

iii) The energy stored in capacitors after the switch has been opened by 50 m sec.

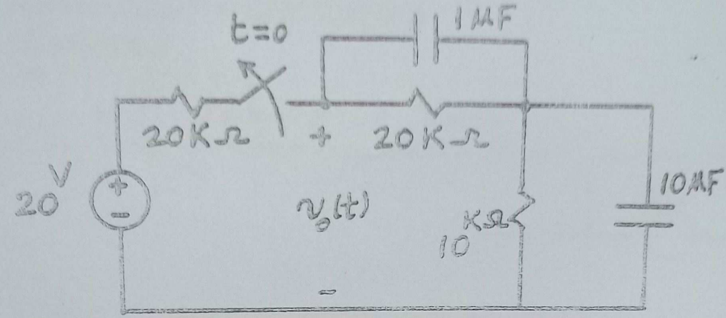


Fig.(4)

b) The switch in the circuit shown in fig.(5) has been closed for a long time. At $t=0$, the switch is opened. Find $v(t)$ for $t \geq 0$

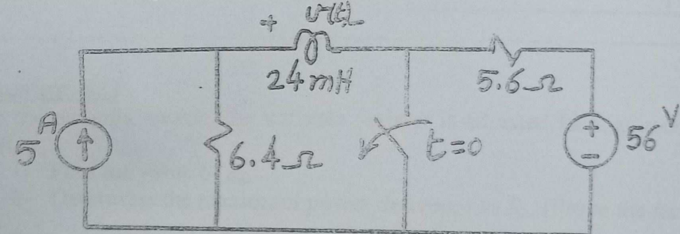


Fig.(5)

(15 marks)

5- For the balanced 3-phase system shown in fig.(6), determine:

a) line currents.

b) line voltages at the load terminals.

c) the values of Δ – connected capacitors bank, connected at the load terminals in order to raise the power factor to 0.95.

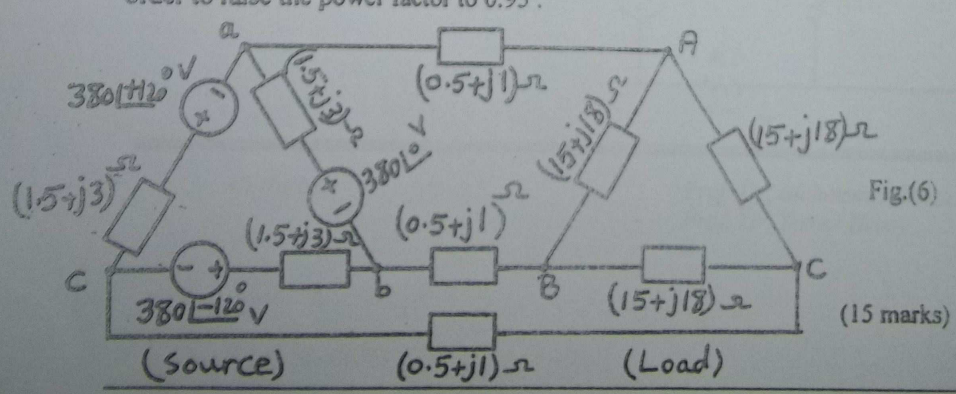


Fig.(6)

(15 marks)

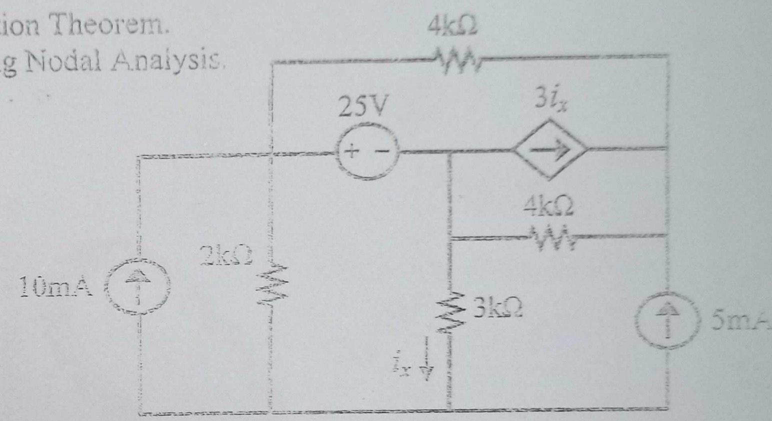
Alexandria University
Faculty of Engineering
Department of Electrical Engineering
EE111 Electrical Circuits
First Year Electrical Engineering Department Students
Mid Term Exam Fall 2010/2011 Time Allowed: 1 Hour

Answer the following Questions

Question (1)

For the circuit shown,

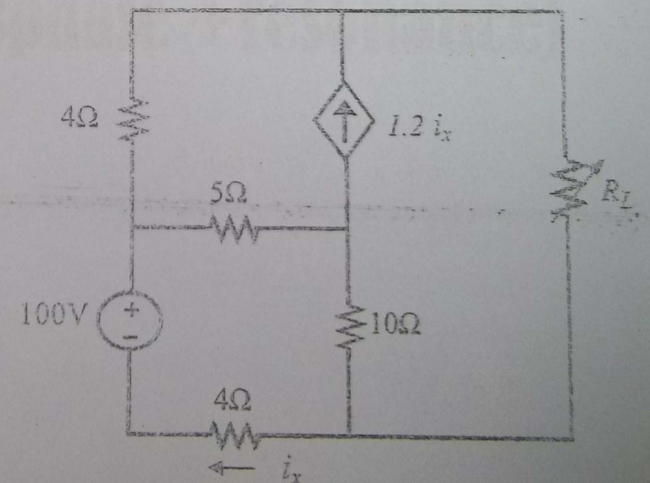
1- Find i_x using Superposition Theorem.
2- Verify your answer using Nodal Analysis.



Question (2)

For the network shown, the variable resistor is adjusted for maximum power transfer to R_L.

1- Find the value of R_L.
2- Determine the maximum power delivered to R_L. (Prove the formula used)



All the best *Prof. Dr. Mohammed Zakareia*

Second

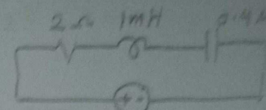
Solved examples (3phase +resonance)

mple 1 Consider the circuit of Fig. 1.

(a) Find the resonant frequency ω_0 and the half-power frequencies, (ω_1, ω_2)

(b) Calculate the bandwidth and quality factor.

(c) Determine the amplitude of current at ω_0, ω_1 and ω_2.



$v(t) = 20 \sin \omega t$

Fig. 1

Solution (a) The resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times10^{-3} \times 0.4\times10^{-6}}} = 50\times10^3 \ r/s = 50 \ Kr/s.$$

The half-power frequencies are:

$$\omega_{1,2} = \mp \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \omega_0^2}$$

$$\therefore \omega_{1,2} = \mp \frac{2}{2\times1\times10^{-3}} + \sqrt{\left(\frac{2}{2\times1\times10^{-3}}\right)^2 + (50\times10^3)^2}$$

$$\therefore \omega_1 = 49009.99 \ r/s \quad , \quad \omega_2 = 51009.99 \ r/s.$$

(b) The bandwidth is

$$\beta = \omega_2 - \omega_1 = 2000 \ r/s$$

$$\left(\text{or, } \beta = \frac{R}{L} = \frac{2}{1\times10^{-3}} = 2000 \ r/s\right).$$

$\therefore$ The, quality factor is

$$Q = \frac{\omega_0}{\beta} = \frac{50\times10^3}{2000} = 25 \quad \text{(dimensionless)}.$$

$$\left(\text{or, } Q = \frac{\omega_0 L}{R} = \frac{50\times10^3 \times 1\times10^{-3}}{2} = 25\right).$$

Note that, since $Q > 10$, this is a high-Q circuit and we can calculate the half-power frequencies as:

$$\omega_{1,2} \simeq \omega_0 \mp \frac{\beta}{2} = 50\times10^3 \mp \frac{2000}{2} = 50\times10^3 \mp 1\times10^3$$

$$\therefore \omega_1 \simeq 49\times10^3 \quad , \quad \omega_2 \simeq 51\times10^3 \ r/s.$$

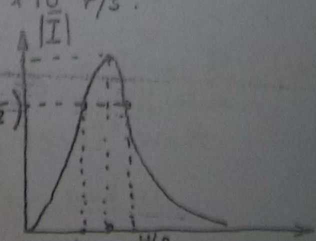
(c) Since $I_p = \frac{V_{max}}{R}$, at ω_0,

$$\therefore I_p = \frac{20}{2} = 10 A$$

At ω_1, or, ω_2:

$$I(\omega_1) = I(\omega_2) = \frac{I_p}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.071 A$$

$(I(\omega_1) = I(\omega_2) = I_p/\sqrt{2})$



Example 2 In the parallel RLC circuit of Fig. 2, find:

(a) ω_o and Q. (b) ω_1 and ω_2.

(c) The power dissipated at ω_o, ω_1 and ω_2.

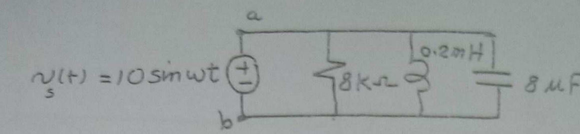


Fig. 2

Solution (a) $\omega_o = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.2 \times 10^{-3} \times 8 \times 10^{-6}}} = 25 \times 10^3$ r/s.

The bandwidth is $\beta = \dfrac{1}{RC}$, and the quality factor is

$Q = \dfrac{\omega_o}{\beta} = \omega_o RC$ $\therefore \beta = \dfrac{1}{8 \times 10^3 \times 8 \times 10^{-6}} = 15.625$ r/s

$\therefore Q = 25 \times 10^3 \times 8 \times 10^3 \times 8 \times 10^{-6} = 1600$

(b) Since $Q > 10$, we can regard this circuit as a high-Q circuit. Therefore, the half power frequencies are approximately given by:

$$\omega_{1,2} = \omega_o \mp \frac{\beta}{2} = 25 \times 10^3 \mp \frac{15.625}{2}$$

$\therefore \omega_1 \simeq 24.992 \times 10^3$ r/s, $\omega_2 \simeq 25.008 \times 10^3$ r/s.

(c) At $\omega = \omega_o$, the circuit is pure resistive, i.e.,

$Z_{ab} = R = 8 \times 10^3 \, \Omega$. (see Fig. 3).

$\therefore i(\omega_o) = \dfrac{10}{8 \times 10^3} \sin \omega_o t$

$\therefore i(\omega_o) = 1.25 \times 10^{-3} \sin(25 \times 10^3 t)$

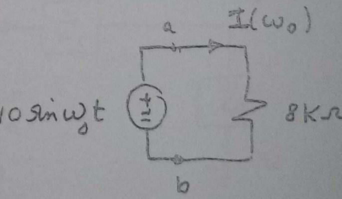


Fig. 3

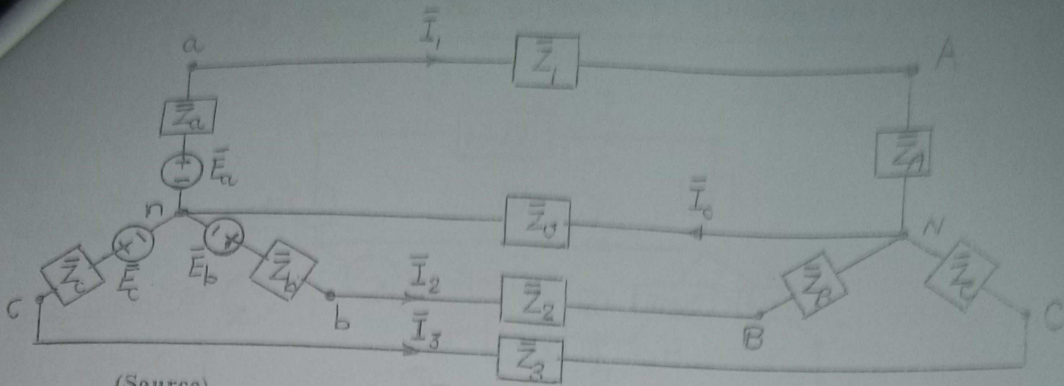
Note that the entire current flows through ~~the~~ 8 KΩ resistor.

$\therefore$ The power dissipated at ω_o is $P_{\omega_o} = I^2_{(\omega_o)\,\text{eff. value}} R$, (Watt)

$\therefore P_{\omega_o} = \left(\dfrac{1.25 \times 10^{-3}}{\sqrt{2}}\right)^2 \times 8 \times 10^3 = 6.25 \times 10^{-3}$ Watt.

At the half-power frequencies ω_1 and ω_2:

$P_{\omega_1} = P_{\omega_2} = \frac{1}{2} P_{\omega_o} = \frac{1}{2} \times 6.25 \times 10^{-3} = 3.125 \times 10^{-3}$ W



(Source) (Transmission lines) (Load)

Fig. 5

(a) What are the conditions for the system to be balanced?

(b) Mention the importance of phase sequence in 3- phase power distribution.

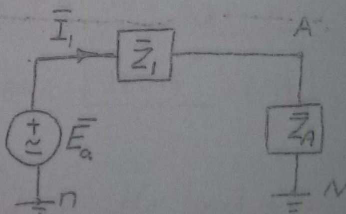
(c) Consider the case of balanced 3- phase source of negligible internal impedence; with $\bar{E}_a = 100\angle 0^\circ$ V, and abc phase sequence. Given that $\bar{z}_1 = \bar{z}_2 = \bar{z}_3 = 1+j2\ \Omega$, and $\bar{z}_0 = 0$. Calculate the line currents and total complex power delivered to each of the following loads:

(i) Balanced load with $\bar{z}_A = \bar{z}_B = \bar{z}_C = 9+j8\ \Omega$.

(ii) Unbalanced load with $\bar{z}_A = 15\ \Omega$, $\bar{z}_B = 10+j5\ \Omega$, and $\bar{z}_C = 6-j8\ \Omega$.

Solution: (i) From the given data, the whole system is balanced. Consequently, we use the per-phase equivalent circuit:

$$\therefore \bar{I}_1 = \frac{\bar{E}_a}{\bar{Z}_1 + \bar{Z}_A} = \frac{100\angle 0^\circ}{1+j2+9+j8}$$



$$\therefore \bar{I}_1 = \frac{10}{\sqrt{2}}\angle -45^\circ$$

$$\therefore \bar{I}_2 = \frac{10}{\sqrt{2}}\angle -165^\circ$$

$$\therefore \bar{I}_3 = \frac{10}{\sqrt{2}}\angle 75^\circ$$

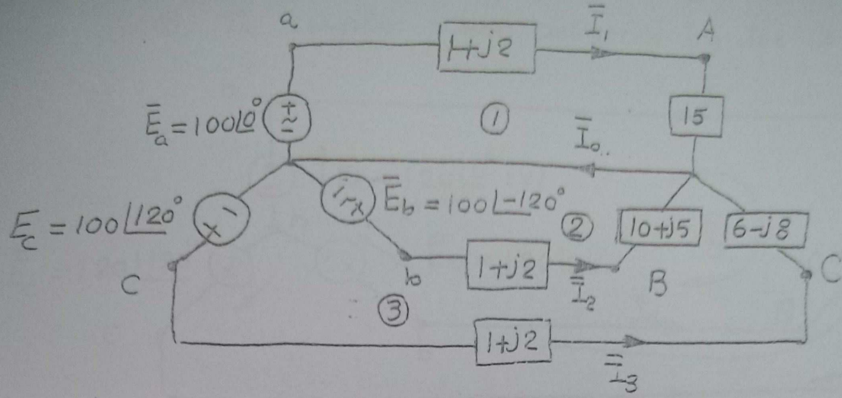
The total complex power delivered to the load is:

$$\bar{S}_\ell = 3\, I_1^2\, \bar{Z}_A = 3\left(\frac{10}{\sqrt{2}}\right)^2 (9+j8)$$

$$\therefore \boxed{\bar{S}_\ell = 1350 + j120}\ \text{(VA)}$$

(N.B.) $\bar{I}_0 = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 0$.

(ii) In this case, the system is unbalanced, as shown below



Apply KVL around each of the three meshes:

Mesh ①: $\bar{E}_a = \bar{I}_1(1+j2+15) = (16+j2)\bar{I}_1 = 100\angle 0^\circ$

$\therefore \bar{I}_1 = \dfrac{100\angle 0^\circ}{(16+j2)} = 6.226\angle -7.125^\circ$, (A)

Mesh ②: $\bar{E}_b = 100\angle -120^\circ = (1+j2+10+j5)\bar{I}_2$

$\therefore \bar{I}_2 = \dfrac{100\angle -120^\circ}{(11+j7)} = 7.67\angle -152.47^\circ$ (A)

Mesh ③: $\bar{E}_c = 100\angle 120^\circ = (1+j2+6-j8)\bar{I}_3$

$\therefore \bar{I}_3 = \dfrac{100\angle 120^\circ}{(7-j6)} = 10.846\angle 160.6^\circ$ (A)

The total complex power delivered to the load is:

$$\bar{S}_\ell = I_1^2 \bar{Z}_A + I_2^2 \bar{Z}_B + I_3^2 \bar{Z}_C$$

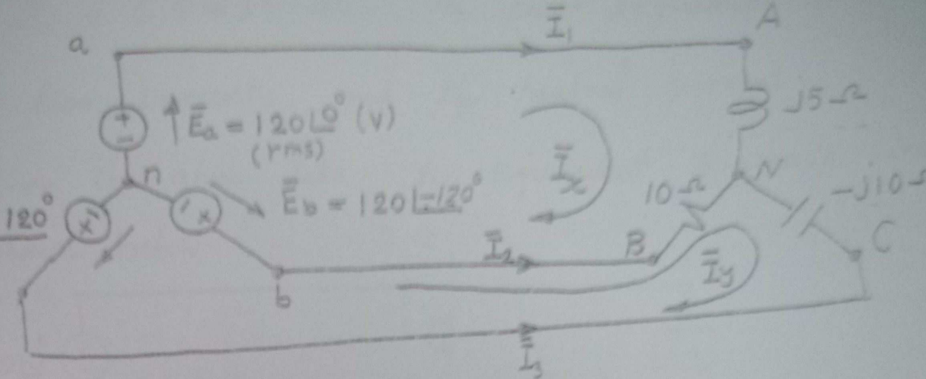
$$= (6.226)^2(15) + (7.67)^2(10+j5) + (10.846)^2(6-j8)$$

$$\therefore \bar{S}_\ell = 1874.9 - j352.75 \text{ (VA)}$$

N.B. $\bar{I}_o = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 \neq 0$.

Example (A)

Find
(a) The line currents.
(b) The complex power absorbed by the load.
(c) The complex power delivered by the source.



Solution Let $\bar{I}_x$ and $\bar{I}_y$ be the 2-mesh currents

Mesh x: $(\bar{E}_a - \bar{E}_b) = (10 + j5)\,\bar{I}_x - 10\,\bar{I}_y = 120\angle 0^\circ - 120\angle -120^\circ$ ...... (1)

Note that: $(\bar{E}_a - \bar{E}_b) = \bar{E}_{ab} = \sqrt{3}\,\bar{E}_a \angle 30^\circ$ = line voltage.

Mesh y: $(\bar{E}_b - \bar{E}_c) = -10\,\bar{I}_x + (10 - j10)\,\bar{I}_y = 120\angle -120^\circ - 120\angle 120^\circ$ ..... (2)

Note that: $(\bar{E}_b - \bar{E}_c) = \sqrt{3}\,\bar{E}_b \angle 30^\circ$ = line voltage.

Solving (1) and (2)

$\bar{I}_x = 56.78\angle 0^\circ$, $\bar{I}_y = 42.75\angle 25^\circ$

Therefore, the 3 line currents are:

$\bar{I}_1 = \bar{I}_x = 56.78\angle 0^\circ$, $\bar{I}_2 = \bar{I}_y - \bar{I}_x = 25.46\angle 135^\circ$, and

$\bar{I}_3 = -\bar{I}_y = -42.75\angle 25^\circ = 42.75\angle -155^\circ$ (A)

$\therefore \bar{S}_{load} = (56.78)^2(j5) + (25.46)^2(10) + (42.75)^2(-j10)$

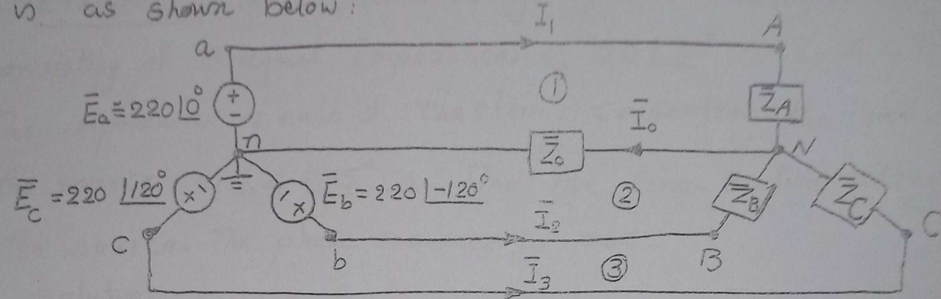
$\therefore \bar{S}_\ell = 6480 - j2156$ (VA) $\therefore \bar{S}_{source} = \bar{S}_\ell$

Check: $\bar{S}_{source} = \sum E_j I_j^* = (120\angle 0^\circ)(56.78\angle 0) + (120\angle -120^\circ)(25.46\angle -135^\circ) + (120\angle 120^\circ)(42.75\angle 155^\circ) = 6480 - j2156 = \bar{S}_\ell$ (correct)

A 3-phase, 4-wire, 380 V source is connected to an unbalanced load having phase impedances of: $\bar{Z}_A = 8+j6\ \Omega$, $\bar{Z}_B = 8-j6\ \Omega$, and $\bar{Z}_C = 5\ \Omega$. The impedance of the neutral wire is $\bar{Z}_o = 0.5+j1\ \Omega$. The phase sequence is abc.

Find the line currents and the neutral current.

<u>Solution</u> From the given data, the equivalent 3-phase circuit is as shown below:



<u>Note that</u>, the given 380 V is the effective (rms) value of line-to-line voltage of the source. Hence, the corresponding value of phase voltage is $\frac{380}{\sqrt{3}} \cong 220$ V.

Using mesh analysis, the 3-mesh equations are:

$$220\angle 0 = \bar{Z}_A \bar{I}_1 + \bar{Z}_o \bar{I}_o \quad \ldots (1)$$

$$220\angle -120^\circ = \bar{Z}_B \bar{I}_2 + \bar{Z}_o \bar{I}_o \quad \ldots (2)$$

$$220\angle 120^\circ = \bar{Z}_C \bar{I}_3 + \bar{Z}_o \bar{I}_o \quad \ldots (3)$$

With, $\bar{I}_o = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 \quad \ldots (4)$

Solving the 4 equations, yields:

$$\boxed{\bar{I}_1 = 22.1\angle -36.3^\circ \text{ A}, \quad \bar{I}_2 = 21.86\angle 82.4^\circ \text{ A}, \quad \bar{I}_3 = 43.45\angle 119.4^\circ \text{ A}, \quad \text{and } \bar{I}_o = 3.12\angle 104^\circ \text{ A}.}$$

<u>Important note:</u> Since $\bar{I}_o \neq 0$, $\therefore \bar{V}_N - \bar{V}_n = \bar{Z}_o \bar{I}_o$. Therefore, if $\bar{V}_n = 0$ (as reference), $\bar{V}_N = \bar{Z}_o \bar{I}_o = -3.14 + j0.76$. (page 7)

3-phase system:

N.B. When we speak of a 3-phase system, the terms voltage, current and power are understood to mean line-to-line voltage, line current and total 3-phase power, respectively.

Example (1): The terminal voltage of a Y-connected load consisting of 3 equal impedances of $20\angle 30^\circ\ \Omega$ is 4.4 KV. The impedance of each of the 3 lines connecting the load to the source is $1.4\angle 75^\circ\ \Omega$. Find the line-to-line voltage at the source. The phase sequence is abc.

Solution:

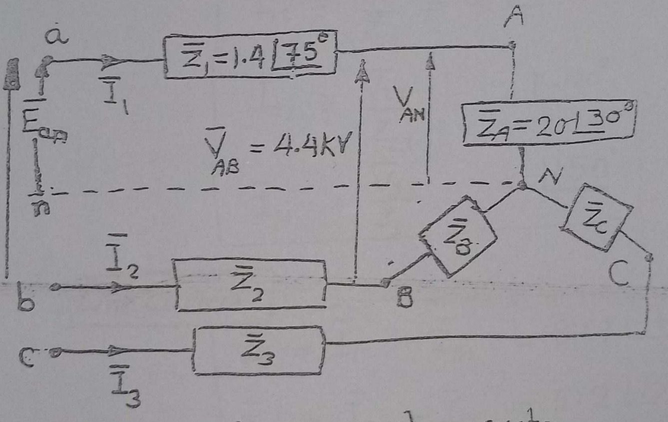


Fig-1 : Given 3-phase system.

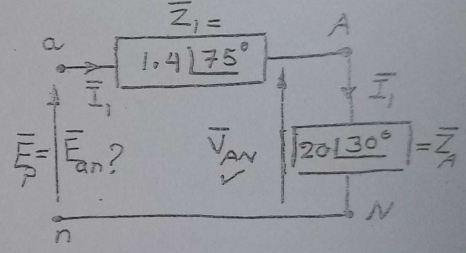


Fig-2 1-phase (or, per-phase) circuit.

Since the given system is balanced, then use the per-phase equivalent circuit of Fig-2.

$\because V_{AB} = 4.4\ KV \qquad \therefore V_{AN} = \frac{4.4}{\sqrt{3}}\ KV \cong 2540\ V$

Let $\bar{V}_{AN} = 2540\angle 0^\circ\ V$, as reference phasor, then Ohm's law gives: $\bar{I}_1 = 2540\angle 0^\circ / 20\angle 30^\circ = 127\angle -30^\circ\ A$, and KVL yields:

$$\bar{E}_{an} = \bar{V}_{AN} + \bar{I}_1\bar{Z}_1 = 2540\angle 0^\circ + (127\angle -30^\circ)(1.4\angle 75^\circ) = 2670\angle 2.7^\circ\ V$$

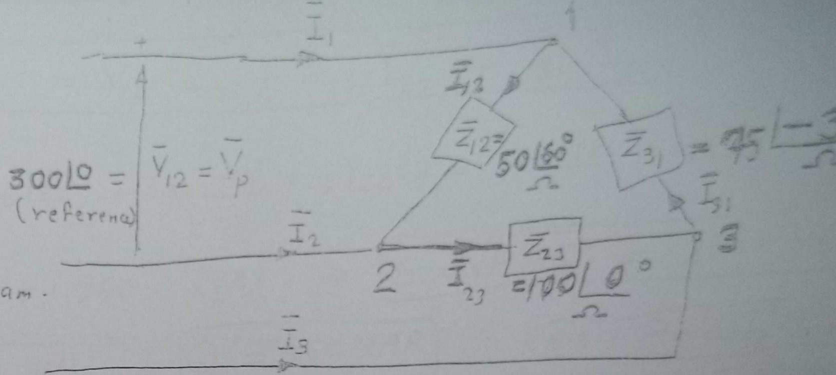
Example (5)

find ① $\bar{I}_{12}, \bar{I}_{23}, \bar{I}_{31}$

② $\bar{I}_1, \bar{I}_2, \bar{I}_3$

③ P, Q, S.

④ Complete phasor diagram.



Solution: For +ve phase sequence, $\overrightarrow{123}$, let

$$\bar{V}_{12} = 300\angle 0 \quad \text{as reference phasor,}$$

$$\therefore \bar{V}_{23} = 300\angle -120^\circ,$$

$$\bar{V}_{31} = 300\angle 120^\circ$$

Phase currents:

$$\bar{I}_{12} = \frac{\bar{V}_{12}}{\bar{Z}_{12}} = 6\angle -60^\circ$$

$$\bar{I}_{23} = \frac{\bar{V}_{23}}{\bar{Z}_{23}} = 3\angle -120^\circ$$

$$\bar{I}_{31} = \frac{\bar{V}_{31}}{\bar{Z}_{31}} = 4\angle 150^\circ$$

Line currents

$$\bar{I}_1 = \bar{I}_{12} - \bar{I}_{31} = 9.67\angle -48^\circ$$

$$\bar{I}_2 = \bar{I}_{23} - \bar{I}_{12} = 5.2\angle 150^\circ$$

$$\bar{I}_3 = \bar{I}_{31} - \bar{I}_{23} = 5\angle 113^\circ$$

Complex power $\tilde{S} = P + jQ$

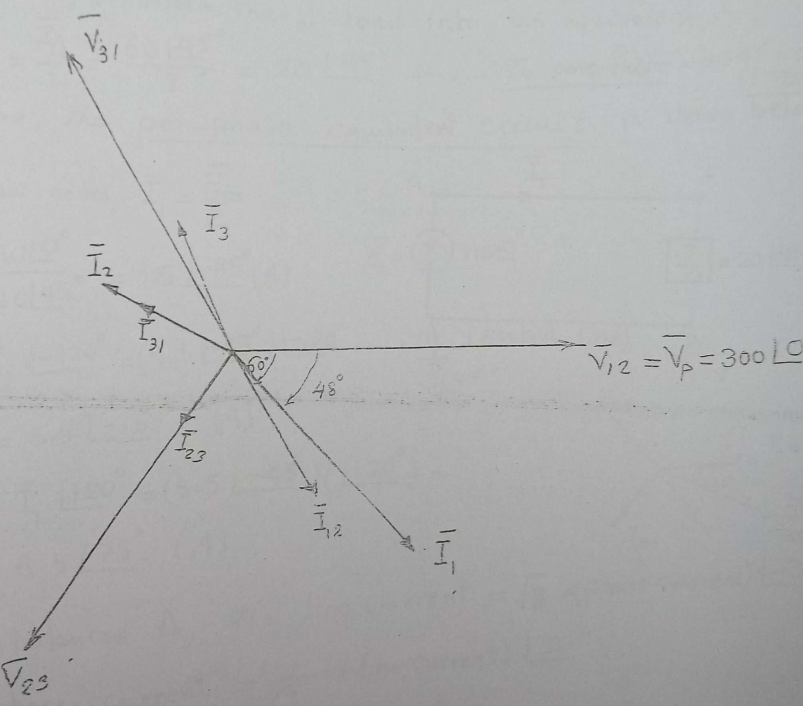
$$\boxed{\tilde{S} = \Sigma V_p I_p = \bar{V}_{12}\,\bar{I}_{12}^{*} + \bar{V}_{23}\,\bar{I}_{23}^{*} + \bar{V}_{31}\,\bar{I}_{31}^{*}}$$

$$= (300\angle 0)(6\angle 60) + (300\angle -120^\circ)(3\angle 120^\circ) + (300\angle 120^\circ)(4\angle 210^\circ)$$

$\therefore \tilde{S} = 2840 + j\,960$ (VA) $\Rightarrow$ $\boxed{S = \sqrt{P^2+Q^2} = 2997\ VA}$

$\therefore$ $\boxed{P = 2840\ W}$, $\boxed{Q = 960\ VAR}$ N.B. $\boxed{P = \Sigma I_p^2 R, \quad Q = \Sigma I_p^2 X}$

phasor diagram



Note that the phase currents and line currents form unbalanced sets.

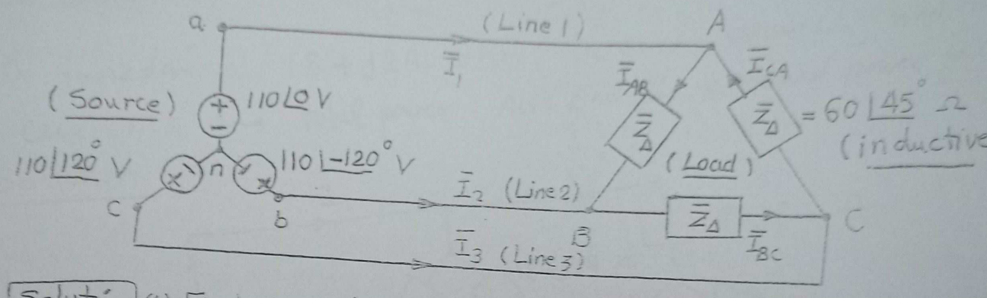
In the balanced 3-phase system shown in fig. ,

(1)- The line currents. [The phase sequence is abc (or ABC, or 123).

(2)- The currents in each phase of the load.

(3)- The complex power delivered to the load.

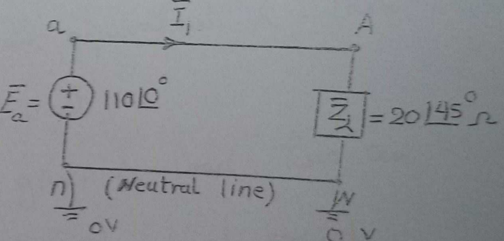
(4)- The power factor of the system



Solution (1) First, transform the Δ-load into an equivalent λ:

$$\therefore \overline{Z}_\lambda = \frac{\overline{Z}_\Delta}{3} = \frac{60\angle 45^\circ}{3} = 20\angle 45^\circ \ \Omega. \quad \therefore \text{The power factor} = \cos 45^\circ = 0.707 \text{ (lagging)}.$$

Therefore, the per-phase equivalent circuit is shown below:



$\therefore$ Ohm's law gives, $\overline{I}_1 = \dfrac{\overline{E}_a}{\overline{Z}_\lambda}$

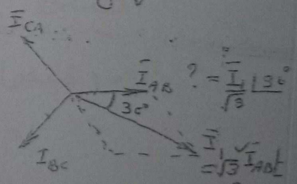
$$\therefore \overline{I}_1 = \frac{110\angle 0^\circ}{20\angle 45^\circ} = 5.5\angle -45^\circ \text{ (A)}$$

$$\therefore \overline{I}_2 = \overline{I}_1 \angle -120^\circ = (5.5\angle -45^\circ)\angle -120^\circ$$

$$\therefore \overline{I}_2 = 5.5\angle -165^\circ \text{ (A)}$$

And, $\overline{I}_3 = \overline{I}_1 \angle 120^\circ = (5.5\angle -45^\circ)(\angle 120^\circ)$

$$\therefore \overline{I}_3 = 5.5\angle 75^\circ \text{ (A)}$$



the line current = $\sqrt{3}$ x (phase current) $\angle -30^\circ$

(2) In a balanced Δ,

$\therefore$ phase current = $\frac{1}{\sqrt{3}}$ x (line current) $\angle 30^\circ$

$$\therefore \overline{I}_{AB} = \frac{\overline{I}_1}{\sqrt{3}}\angle 30^\circ = \frac{(5.5\angle -45^\circ)\angle 30^\circ}{\sqrt{3}} = 3.18\angle -15^\circ \text{ (A)}$$

$$\therefore \overline{I}_{BC} = \overline{I}_{AB}\angle -120^\circ = (3.18\angle -15^\circ)\angle -120^\circ = 3.18\angle -135^\circ \text{ (A)},$$

$$\overline{I}_{CA} = \overline{I}_{AB}\angle 120^\circ = (3.18\angle -15^\circ)\angle 120^\circ = 3.18\angle 105^\circ \text{ (A)}$$

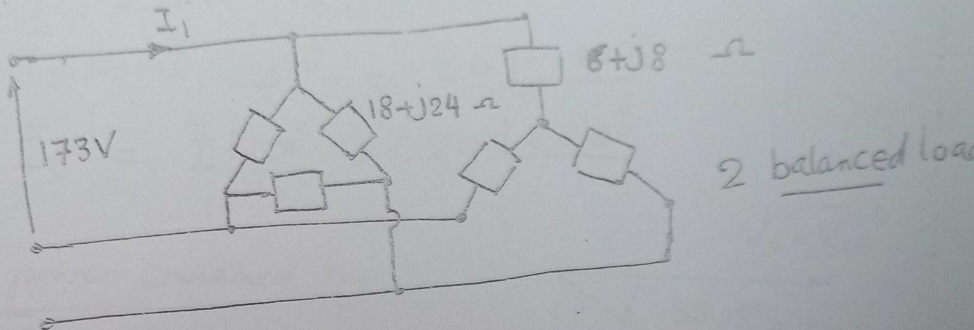
(3) Total 3-phase complex power $\overline{S} = 3\, I_1^2\, \overline{Z}_\lambda = 3(5.5)^2(20\angle 45^\circ) = 1815\angle 45^\circ$ (VA)

(page 5)

Example (6) **Parallel loads:**

A 3-phase, 3-wire supply of 173 V supplies two balanced 3-phase loads; one Y-connected with each branch impedance 6 + j8 ohm, and the other Δ-connected with impedance 18 + j24 ohm in each branch. Find the total line currents and total power. Also find the combined power factor.

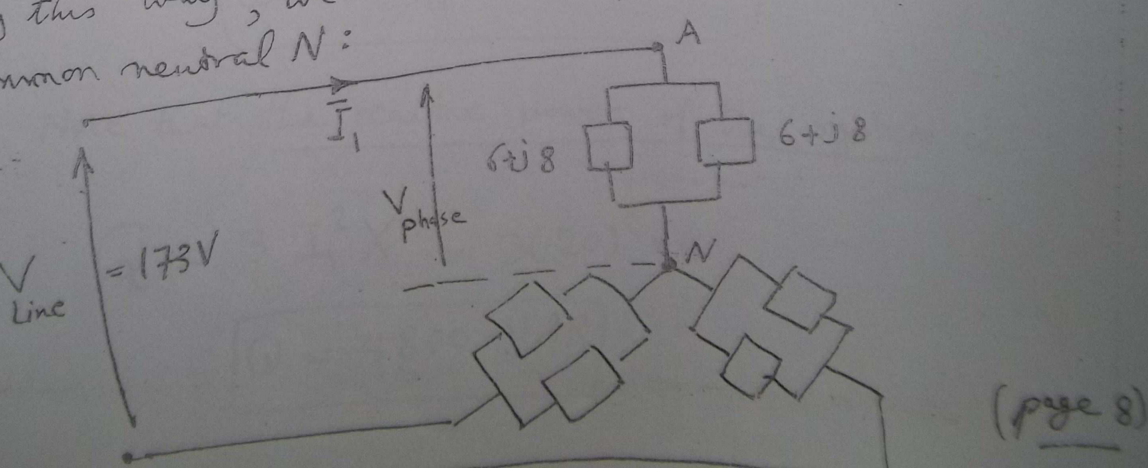
Solution:



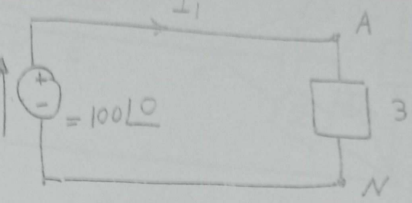
First, convert the Δ-load to an equivalent λ. Then use the per phase equivalent circuit.

The equivalent Y-load of the given Δ-load, has an impedance $\dfrac{18+j24}{3} = 6+j8\ \Omega$, in each branch.

By this way, we have two parallel λ-loads, with a common neutral N:



Then use the per phase equivalent circuit to find the line current, with $V_{phase} = \frac{173}{\sqrt{3}} \cong 100\ V$. Let $\overline{V}_p = 100\angle 0$



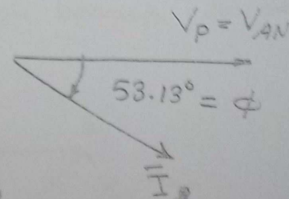
$\overline{V}_{AN} = \overline{V}_p$

Line Currents:

$$\therefore \overline{I}_1 = \frac{\overline{V}_p}{3+j4} = \frac{100\angle 0}{5\angle 53.1^\circ} = 20\angle -53.13^\circ,$$

$$\overline{I}_2 = \overline{I}_1 \angle -120^\circ = 20\angle -173.13^\circ$$

$$\overline{I}_3 = \overline{I}\angle 120^\circ = 20\angle 66.87^\circ$$



The total power consumed by the two loads is

$$P = 3 I_1^2 \times 3 = 3(20)^2 \times 3$$

$$\therefore \boxed{P = 3600\ \text{Watt}}$$

The power factor of the system is

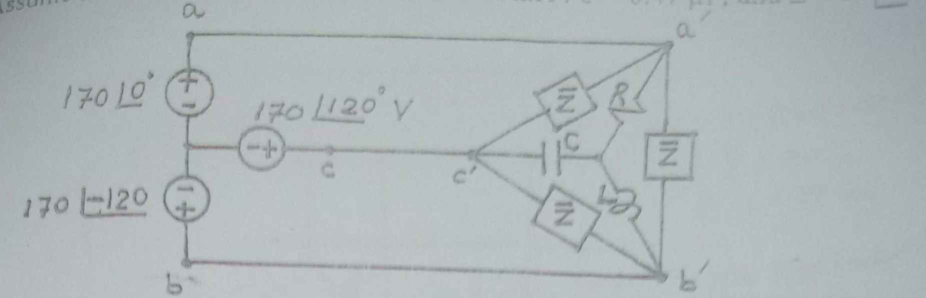
$$\boxed{\cos\phi = \cos 53.13 = 0.6}$$ (lagging P.F.)

Note that the reactive power of the system is

$$Q = 3 I_1^2 X = 3(20)^2 \times 4$$

$$\therefore \boxed{Q = 4800\ \text{VAR}}$$

...nd the line currents and the total power consumed by the load in the system shown in ...3. Assume that : f = 400 Hz, R = 100 Ω, L = 100 mH, C = 0.47 μF, and $\bar{Z} = 50\angle 37^\circ\ \Omega$



(Source) (Load) Fig.3

$\boxed{\text{Solution}}$: $\omega = 2\pi f = 800\pi$ r/s.

$\bar{Z}_C = -j\dfrac{1}{\omega C} = -j846.6\ \Omega$, $\bar{Z}_L = j\omega L = j251.33\ \Omega$

The best way to solve this problem is to transform the unbalanced $\curlywedge$ into an equivalent Δ, as shown in Fig.(a); Then, replace each pair of parallel branches by a single branch to obtain an equivalent Δ, as shown in Fig.(b).

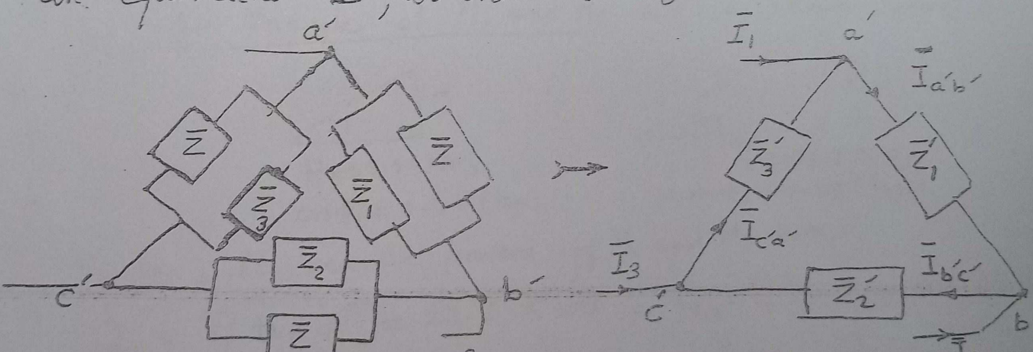


Fig.(b)

$\bar{Z} = 50\angle 37^\circ\ \Omega$,

Fig.(a): $\bar{Z}_2 = 2209.46\angle -15.6^\circ$,

$\bar{Z}_1 = 261\angle 74.4^\circ$, $\bar{Z}_3 = 879.1\angle 74.4^\circ$.

$\bar{Z}_1' = 43.2\angle 42.8^\circ$,

$\bar{Z}_2' = 49.3\angle 36^\circ$,

$\bar{Z}_3' = 52.3\angle 35^\circ$.

The 3 line voltages are, $\bar{V}_{a'b'} = 170\sqrt{3}\angle 30^\circ$, $\bar{V}_{b'c'} = 170\sqrt{3}\angle -90^\circ$, and $\bar{V}_{c'a'} = 170\sqrt{3}\angle 150^\circ$. Therfore, the phase currents in Fig.(b) are:

$\bar{I}_{a'b'} = \dfrac{\bar{V}_{a'b'}}{\bar{Z}_1'} = 6.82\angle -12.8^\circ$, $\bar{I}_{b'c'} = 6\angle -126^\circ$, $\bar{I}_{c'a'} = 5.6\angle 115^\circ$.

The line currents are : $\bar{I}_1 = \bar{I}_{a'b'} - \bar{I}_{c'a'} = 11.2\angle -36^\circ$, $\bar{I}_2 = 10.7\angle -161^\circ$, $\bar{I}_3 = 10\angle 183.3^\circ$. Therefore, P_{load} = Real part of $[I_{a'b'}^2\bar{Z}_1' + I_{b'c'}^2\bar{Z}_2' + I_{c'a'}^2\bar{Z}_3']$

Or, $P_{load} = 1474.3 + 1435.8 + 1343.5 = 4253.6$ Watt.

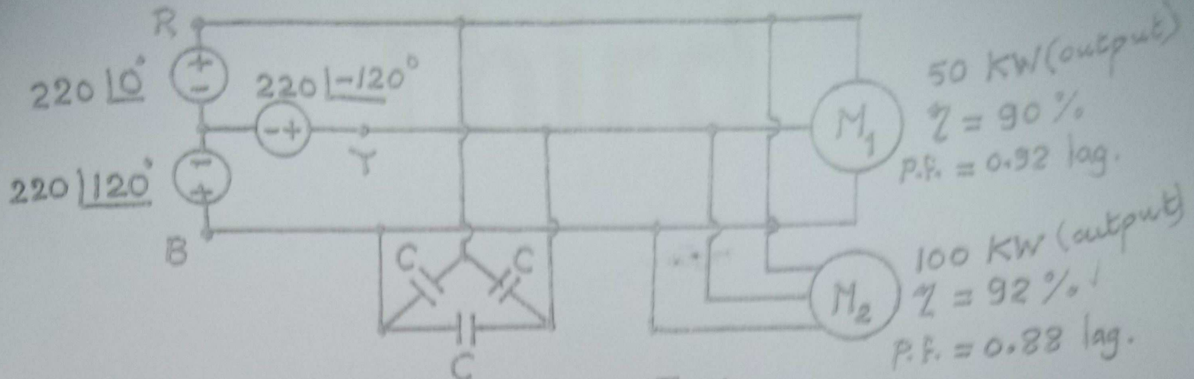
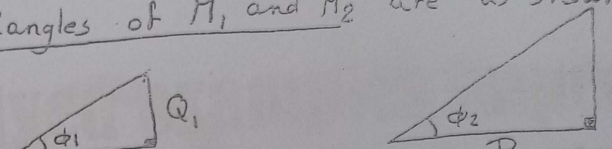


Fig. 4

Solution : First determine the input power of the two loads M_1 and M_2, using the relation : $\eta = \dfrac{P_{output}}{P_{input}}$

$\therefore$ For M_1, M_2: $P_1 = \dfrac{50}{0.9} = 55.56$ KW , $P_2 = \dfrac{100}{0.92} = 108.7$ KW

The power triangles of M_1 and M_2 are as shown below :



$P_1 = 55.56$ KW, $\cos\phi_1 = 0.92$ (lag.).
$P_2 = 108.7$ KW, $\cos\phi_2 = 0.88$ (lag.).

Therefore, the reactive powers of M_1 and M_2 are:

$Q_1 = P_1 \tan\phi_1 = 23.7$ KVAR , $Q_2 = P_2 \tan\phi_2 = 58.67$ KVAR.

The total reactive power of the two loads is

$$Q = Q_1 + Q_2 = 82.37 \text{ KVAR}$$

To increase the power factor of the system to unity, the reactive power of the 3 capacitors must equal to $Q = 82.37$ KVAR. Consequently, the reactive power of each capacitor is $Q_c = \dfrac{Q}{3} = 27.46$ KVAR.

The capacitance is : $C = \dfrac{Q_c}{\omega V_c^2}$, where $\omega = 2\pi f$, and $V_c = 220\sqrt{3}$ (Volt) . $\therefore C = 27.46 \times 10^3 / 2\pi \times 50 (220\sqrt{3})^2$

(page 11)

the system shown in fig. 4, determine the value of C to raise the power factor to ... The supply frequency is 50 Hz. (M_1 and M_2 are two motors).

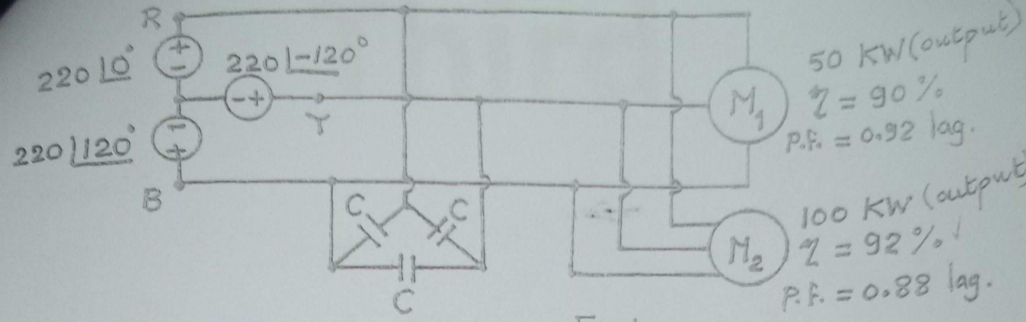
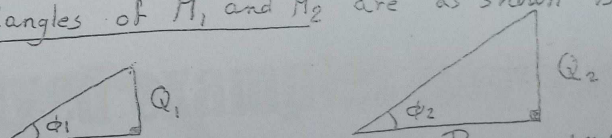


Fig. 4

Solution: First determine the input power of the two loads M_1 and M_2, using the relation: $\eta = \dfrac{P_{output}}{P_{input}}$

$\therefore$ For M_1, M_2: $P_1 = \dfrac{50}{0.9} = 55.56$ KW, $P_2 = \dfrac{100}{0.92} = 108.7$ KW

The power triangles of M_1 and M_2 are as shown below:



$P_1 = 55.56$ KW, $\cos\phi_1 = 0.92$ (lag.).

$P_2 = 108.7$ KW, $\cos\phi_2 = 0.88$ (lag.).

Therefore, the reactive powers of M_1 and M_2 are:

$Q_1 = P_1 \tan\phi_1 = 23.7$ KVAR, $Q_2 = P_2 \tan\phi_2 = 58.67$ KVAR.

The total reactive power of the two loads is

$$Q = Q_1 + Q_2 = 82.37 \text{ KVAR}$$

To increase the power factor of the system to unity, the reactive power of the 3 capacitors must equal to $Q = 82.37$ KVAR. Consequently, the reactive power of each capacitor is $Q_c = \dfrac{Q}{3} = 27.46$ KVAR.

The capacitance is: $C = \dfrac{Q_c}{\omega V_c^2}$, where $\omega = 2\pi f$, and $V_c = 220\sqrt{3}$ (Volt).

$$\therefore C = 27.46 \times 10^3 / 2\pi \times 50 (220\sqrt{3})^2$$

(page 11)

Third

Solved examples impedance

1. Consider the phase shifting circuit shown in Fig.1. The source frequency is 1000 Hz. The values of R and C are always adjusted so that the magnitude of their total impedance is 5000Ω.
Determine the values of R and C which produce a phase shift of 30° between $V_s(t)$ and $V_o(t)$. Show whether $V_o(t)$ lags or leads $V_s(t)$.

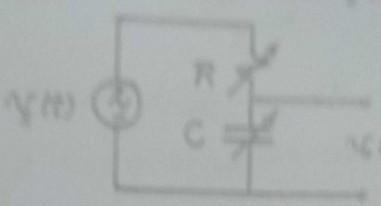
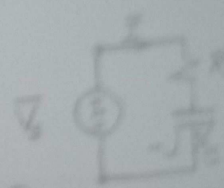
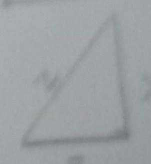


Fig.1





The impedance of the circuit is:

$$\bar{Z} = R - jX_c$$

$$|\bar{Z}| = Z = \sqrt{R^2 + X_c^2} = 5000 \quad \text{.....} \quad (1)$$

From Fig.1:

$$\frac{\bar{V}_o}{\bar{V}_s} = \frac{\cancel{\bar{I}}(-jX_c)}{\cancel{\bar{I}}(R - jX_c)} = \frac{X_c \angle -90^\circ}{\sqrt{R^2+X_c^2}\ \angle -\tan^{-1}\frac{X_c}{R}}$$

$$\therefore \frac{\bar{V}_o}{\bar{V}_s} = \frac{X_c}{\sqrt{R^2+X_c^2}} \angle \tan^{-1}\left(\frac{X_c}{R}\right) - 90^\circ$$

$$\therefore \text{phase shift} = \tan^{-1}\left(\frac{X_c}{R}\right) - 90^\circ = -30^\circ$$

$$\therefore \tan^{-1}\left(\frac{X_c}{R}\right) = 60^\circ$$

$$\therefore X_c = \sqrt{3}\,R \quad \text{.....} \quad (2)$$

Solving (1) and (2)

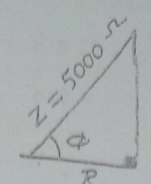
$$\therefore \boxed{R = 2500\ \Omega}, \text{ and } \quad X_c = \frac{1}{\omega C} = 4330.13\ \Omega$$

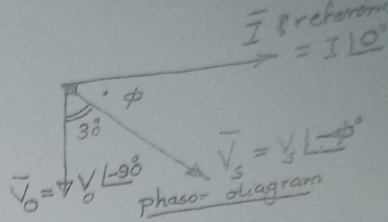
$$\omega = 2\pi \times 1000 \ \text{r/s}$$

$$\therefore \boxed{C \cong 0.037\ \mu F}$$

Another simple solution, using phasor diagram and impedance triangle :

Impedance triangle





phasor diagram

From the phasor diagram, $\phi = 90^\circ - 30^\circ = 60^\circ$

$\therefore\ R = Z\cos\phi = 5000\cos 60^\circ = 2500\ \Omega$,

$X_C = Z\sin\phi = 5000\sin 60^\circ = 5000 \times \frac{\sqrt{3}}{2} = 4330.13\ \Omega$, as before.

$\boxed{\text{N.B.}}$ $\bar{V}_o$ lags $\bar{V}_s$ by 30°.

---

2. In the circuit of Fig.2, find:
 a- The value of R if $\bar{I}$ is to lag behind $\bar{V}_s$ by 90°.
 b- The value of R which will draw maximum power from the circuit.

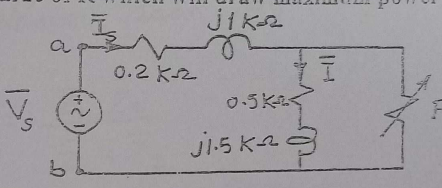


Fig.2

a) The impedance seen by the source is :

$$\bar{Z}_{ab} = \bar{Z}_1 + \frac{R\bar{Z}_2}{(R+\bar{Z}_2)}\ k\Omega,\quad \text{where } \bar{Z}_1 = 0.2 + j1,\ \bar{Z}_2 = 0.5 + j1.5$$

$$\therefore\ \bar{I}_s = \frac{\bar{V}_s}{\bar{Z}_{ab}} = \frac{V(R+\bar{Z}_2)}{\bar{Z}_1\bar{Z}_2 + R(\bar{Z}_1+\bar{Z}_2)},\quad \text{where } \bar{V}_s = V\angle 0$$

Using the current divider,

$$\therefore\ \bar{I} = \bar{I}_s\,\frac{R}{(R+\bar{Z}_2)} = \frac{VR}{\bar{Z}_1\bar{Z}_2 + R(\bar{Z}_1+\bar{Z}_2)}$$

$$\therefore\ \bar{I} = \frac{VR}{(0.7R - 1.4) + j(2.5R + 0.8)}$$

N.B. From the last eq.! $\bar{I}$ lags $\bar{V}_s$ by 90° ... denominator's real part equals zero, i.e., $(0.7R - 1.4) = 0$ ... k

$$\therefore \angle \bar{I} = -\tan^{-1}\left(\frac{2.5R+0.8}{0.7R-1.4}\right) = -90^\circ$$

$$\therefore \tan\left(\frac{2.5R+0.8}{0.7R-1.4}\right) = \infty$$

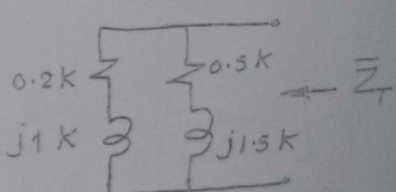
$$\therefore 0.7R - 1.4 = 0$$

$$\therefore \boxed{R = 2\ K\Omega}$$

(b) Using Thevenin's theorem ; the value of R for maximum power transfer is :

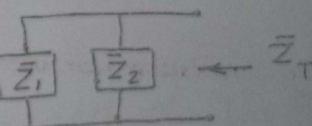
$$R = \sqrt{R_T^2 + X_T^2} = |\bar{Z}_T|$$

The value of $\bar{Z}_T$ is determined by short circuiting the independent source $\bar{V}_s$, and then finding the impedance seen by the load :



$$\therefore \bar{Z}_T = \frac{\bar{Z}_1\ \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$\therefore \bar{Z}_T = 0.1513 + j0.602\ K\Omega$$



$$\therefore \boxed{R = 0.62\ K\Omega}$$

Note that , $\bar{Z}_T$ may be determined by another 2 methods

$$\bar{Z}_T = \frac{\bar{V}_T}{\bar{I}_{sc}}$$

or, $$\bar{Z}_T = \frac{\bar{V}_{ex}}{\bar{I}_{ex}}$$

...find the value of C for which the current drawn from the source is minimum, and find the value of that current. Sketch a phasor diagram representing the source voltage and various currents.

$v_s(t) = 100\sqrt{2} \sin 100\pi t$

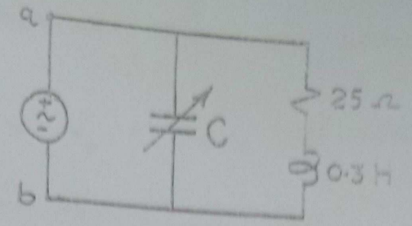


Fig.3

The circuit in the phasor domain is shown in Fig.(1-3)

∴ The admittance seen by the source is :

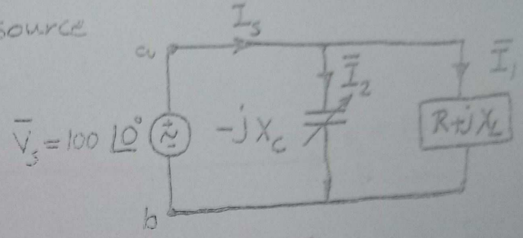


Fig.(1-3)

$$\bar{Y}_{ab} = \frac{1}{-jX_c} + \frac{1}{R+jX_L},$$

with $X_c = \frac{1}{\omega C}$, $X_L = \omega L$,

$\omega = 100\pi$ r/s

$$\therefore \bar{Y}_{ab} = \frac{R}{(R^2 + \omega^2 L^2)} + j\omega\left[C - \frac{L}{(R^2+\omega^2 L^2)}\right]$$

Therefore, the supply current is minimum when $\bar{Y}_{ab}$ is minimum, i.e., $\bar{Z}_{ab}$ is maximum

∴ $\bar{Y}_{ab}$ is minimum if the imaginary part of $\bar{Y}_{ab}$ equal zero. That is, the source current is in phase with $\bar{V}_s$.

$$\therefore C = \frac{L}{R^2+\omega^2 L^2} = \frac{0.3}{25^2 + (100\pi)^2 (0.3)^2}$$

$$\therefore \boxed{C \cong 31.583 \ \mu F}$$

∴ The minimum value of $\bar{Y}_{ab}$ is :

$$Y_{min} = \frac{R}{R^2+\omega^2L^2} = \frac{25}{25^2+(100\pi)^2(0.3)^2} = 2631.92\mu\mho$$

and the minimum value of source current is

$$I_{s\,min} = V_s * Y_{min} = 100 * 2631.92 \times 10^{-6}$$

$$\therefore \boxed{I_{s\,min} = 0.2632\ A}$$

The phasor diagram, is shown in Fig.(2-3)

N.B.:

$$\bar{I}_s = \bar{I}_1 + \bar{I}_2$$

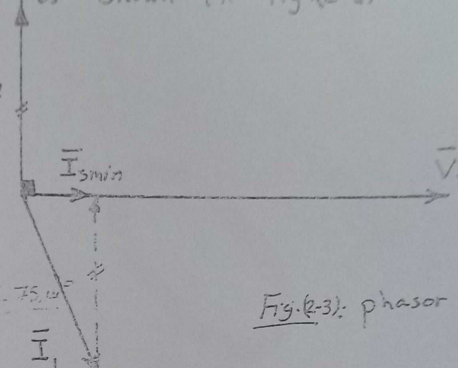


Fig.(2-3): phasor diagram.

Another simple solution for question 3:

From the phasor diagram, it is clear that the source current is minimum when the source power factor is unity. To achieve this objective, the reactive power of the parallel capacitor must equal to the load reactive power (here the load is formed of R, L), as shown in Fig.(3-3); where,

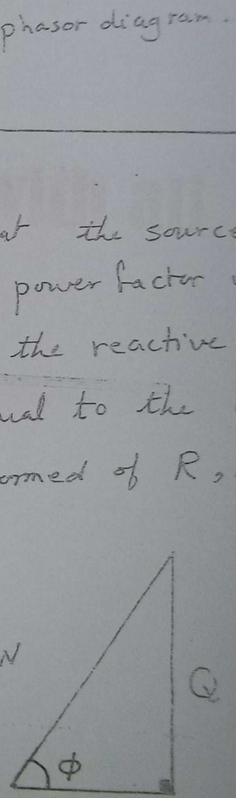


Fig.(3-3): power triangle

$$P = I_1^2 R = \frac{V_s^2}{(R^2+\omega^2L^2)} * R = 26.3\,W$$

$$\therefore Q = P\tan\phi \quad , \quad \phi = \tan^{-1}\frac{X_L}{R}$$

$$\therefore Q = 99.1\ VAR = Q_c = V_s^2 * \omega C$$

$$\therefore \boxed{C \simeq 31.5\ \mu F}$$

Fourth

Circuit analysis solved with all methods

Ex. In the circuit shown in Fig. 1, find the value of i using

[1] Kirchhoff's laws. [2] Nodal analysis. [3] Mesh analysis.

[4] Source transformation. [5] Superposition. [6] Thevenin's method.

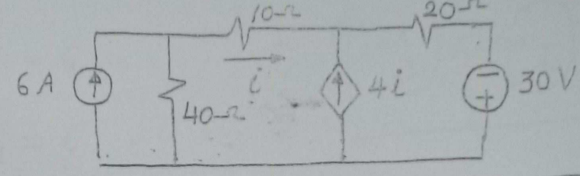


Fig. 1

Solution: [1] By Kirchhoff's laws; (KCL and KVL):

First, distribute the currents in different branches, as shown in Fig. 2.

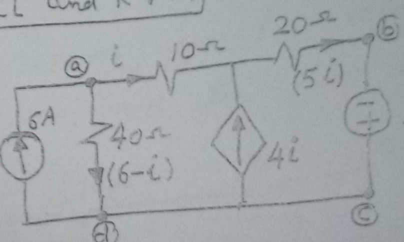


Fig. 2

Then, apply KVL around the loop abcda ($\Sigma V_{rise} = 0$)

$\therefore \quad 30 = 10i + 20(5i) - 40(6-i)$ $\quad \therefore \boxed{i = 1.8\ A}$

---

[2] Using nodal analysis: select lower node as reference (0V), then, write the voltages of the remaining nodes, as shown in Fig. 3.

Now, you have 3 unknowns (i, V_1, V_2).

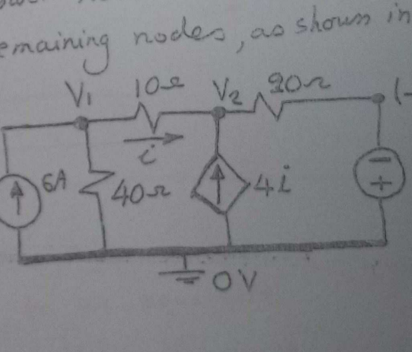


Fig. 3

Node 1: $-6 + \dfrac{(V_1 - 0)}{40} + i = 0$

$\therefore \quad V_1 = 240 - 40i \quad \ldots (1)$

Node 2: $-i - 4i + \dfrac{V_2 - (-30)}{20} = 0$ $\quad \ldots (2)$

$\therefore \quad V_2 = 100i - 30$

Constraint equation: $V_1 - V_2 = 10i$, (Ohm's law) $\quad \ldots (3)$

Solving (1), (2) and (3)

$\therefore \boxed{i = 1.8\ A}$

[3] **Using mesh analysis**: Assume the 3 mesh-currents I_1, I_2 and I_3 as shown in Fig.4. Now, you have 4 unknowns (I_1, I_2, I_3 and i)

The 3 mesh-equations are:

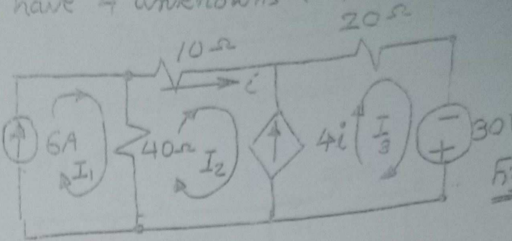


Fig.4

Mesh 1: $I_1 = 6$ ...... ①

Mesh 2: $I_2 = i$ ...... ②

Mesh 3: $I_3 - I_2 = 4i$ ... ③

To write the 4^{th} equation, imagine that you have removed all current sources; by this way you have the loop abcda, shown in Fig.5.

Apply KVL around this loop, you get:

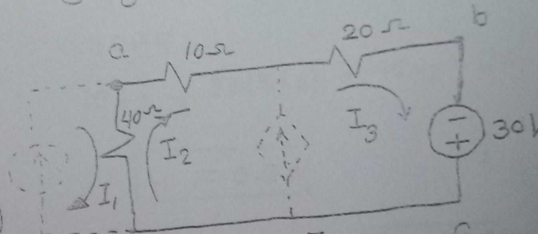


Fig.5

$$30 = 10\,I_2 + 20\,I_3 + 40(I_2 - I_1)$$

$$\therefore\ 5\,I_2 - 4\,I_1 + 2\,I_3 = 3 \quad \text{.... ④}$$

Solve ① → ④, you obtain: $\boxed{i = 1.8\ A}$

---

[4] **By source transformation**:

First transform the (30 V, 20 Ω) to an equivalent current source $\frac{30}{20} = 1.5$ A, in parallel with 20 Ω, as shown in Fig.6. Then combine the // (4i) and 1.5 A sources into a single source (4i − 1.5) A, as shown in Fig.7.

Finally, transform the (6 A, 40 Ω) and [(4i − 1.5) A, 20 Ω] to equivalent (240 V, 40 Ω) and [(80i − 30) V, 20 Ω], Fig.8.

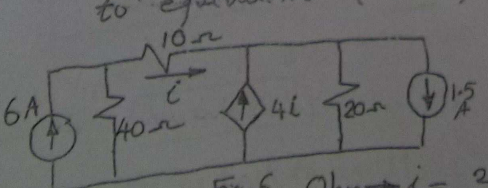


Fig.6

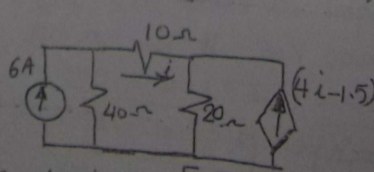
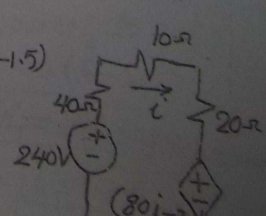


Fig.7



$\therefore$ Ohm → $i = \dfrac{240-(80i-30)}{70}$ ⇒ $\boxed{i = 1.8\,A}$

(page 2)

ⓐ 6A-ON, and 30V-OFF

The corresponding circuit is shown in Fig. 9:

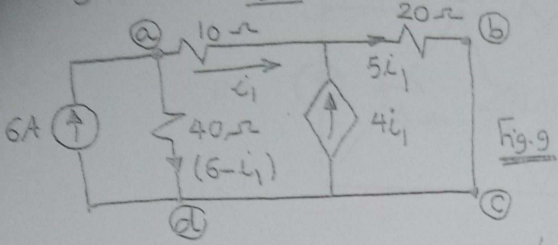


Fig. 9

∴ To find i_1, apply KVL around the loop ⓐⓑⓒⓓⓐ:

$$\therefore 10i_1 + 5i_1(20) - 40(6-i_1) = 0$$

$$\therefore \boxed{i_1 = 1.6\,A}$$

ⓑ 30V-ON, and 6A OFF

The corresponding circuit is shown in Fig. 10:

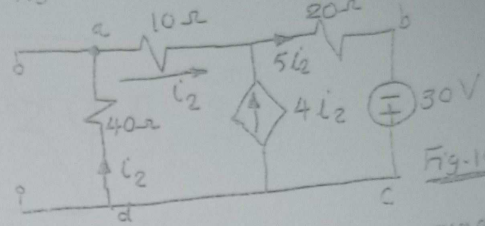


Fig. 10

To find i_2, apply KVL around abcda:

$$\therefore 10i_2 + 20(5i_2) + 40i_2 = 30$$

$$\therefore \boxed{i_2 = 0.2\,A}$$

Finally, superpose i_1 and i_2, i.e., $i = i_1 + i_2 \Rightarrow \boxed{i = 1.8\,A}$

6 Using Thevenin's theorem: To find i through 10Ω, Thevenin's equivalent circuit is shown in Fig. 11, where:

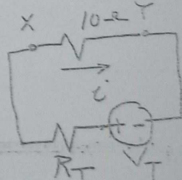


Fig. 11

V_T = open-circuit voltage between X and Y,

$\boxed{R_T = \dfrac{V_T}{I_{s.c.}}}$, where I_{sc} = short-circuit current through XY.

V_T and I_{sc} are determined using Fig. 1, as in Figs. 12 and 13, respectively;

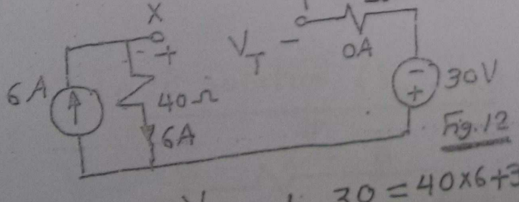


Fig. 12

$$V_T = V_{40\Omega} + 30 = 40\times6+30$$

$$\boxed{V_T = 270\,V}$$

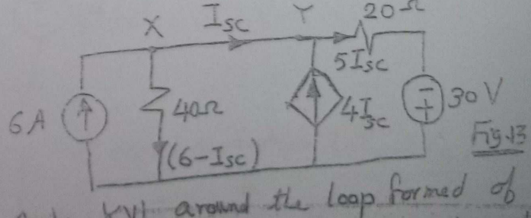


Fig. 13

Apply KVL around the loop formed of 20Ω, 40Ω and the 30V source, to get:

$$30 = 20(5I_{sc}) - 40(6-I_{sc})$$

$$\therefore \boxed{I_{sc} = \frac{27}{14}\,A} \quad \therefore \boxed{R_T = 140\,\Omega}.$$

Now, go back to Fig. 11, $\therefore\ i = \dfrac{V_T}{(R_T+10)} = \dfrac{270}{(140+10)} \Rightarrow \boxed{i = 1.8\,A}$

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Solution of mid-term exam

1. For the circuit shown in fig.(1), it is desired that voltage drop between a and b is equal to 3 volts by adjusting the value of (I_s). Find I_s to achieve the desired value of voltage drop.

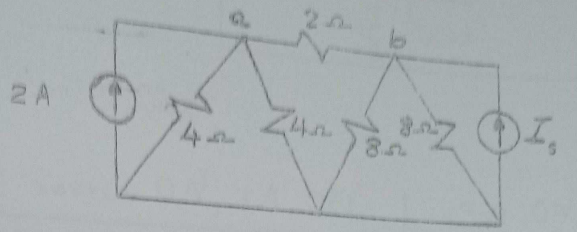
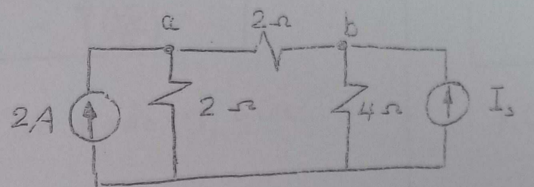


Fig.(1)

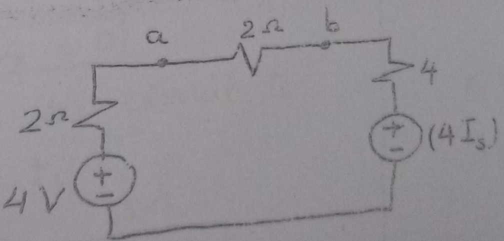
The problem has 2 possible solutions, and can be solved using source transformation:

∵ $4 // 4 \rightarrow 2\Omega$, $8 // 8 \rightarrow 4\Omega$

∴ The circuit is reduced to:

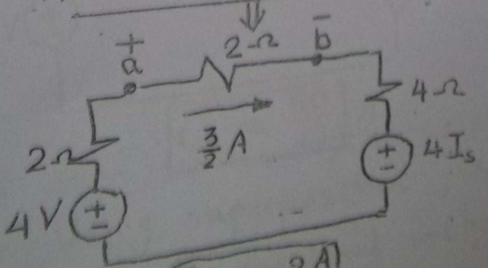


⇓



First solution ($V_a > V_b$):

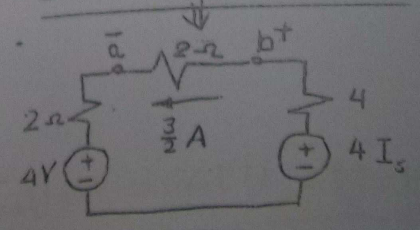
⇓



∴ $\boxed{I_s = -2\,A}$

Second solution ($V_b > V_a$):

⇓



∴ $\boxed{I_s = 4\,A}$

Use superposition theorem to find v_x and i_x in the circuit of Fig.(2)

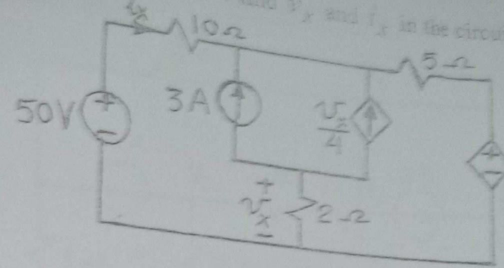
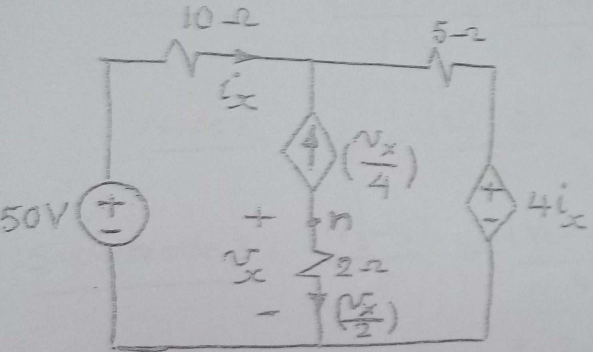


Fig.(2)

50-V Source ON, 3A OFF:

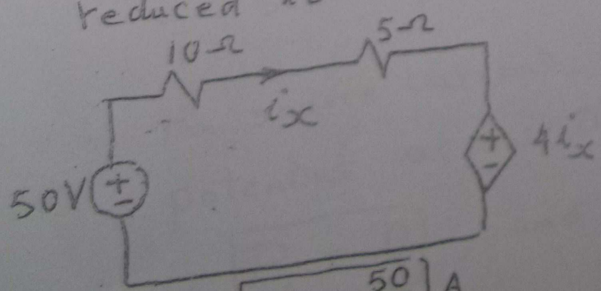


∴ KCL, at node n, gives:

$$\frac{v_x}{4} + \frac{v_x}{2} = 0$$

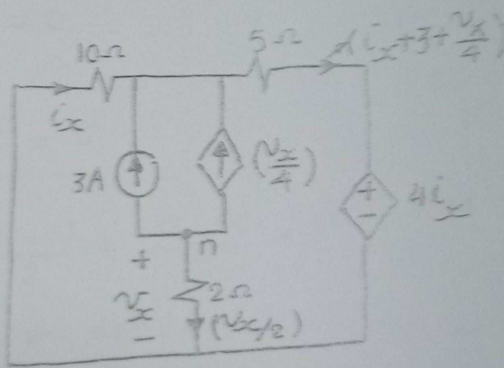
∴ $\boxed{v_x = 0}$

Therefore, the circuit is reduced to



∴ $\boxed{i_x = \frac{50}{19}}$ A

3-A ON, 50V OFF:



∴ KCL, at node n, gives:

$$3 + \frac{v_x}{4} + \frac{v_x}{2} = 0$$

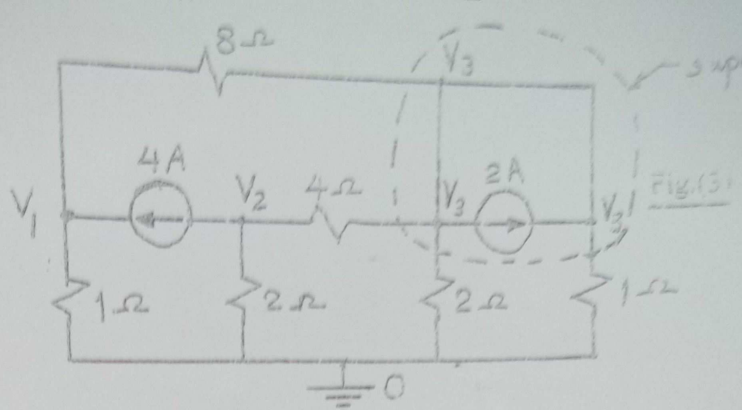
∴ $\boxed{v_x = -4}$ V

KVL, around outer loop, gives:

$$10i_x + 5\left(i_x + 3 + \frac{v_x}{4}\right) + 4i_x = 0$$

∴ $\boxed{i_x = -\frac{10}{19}}$ A

Using superposition, the resultant current $\boxed{i_x = \frac{50}{19} - \frac{10}{19} = \frac{40}{19}}$ A, the resultant voltage is $\boxed{v_x = 0 - 4 = -4}$ V.



Node 1: $\frac{(V_1-0)}{1} + \frac{(V_1-V_3)}{8} - 4 = 0$ (1)

Node 2: $4 + \frac{(V_2-0)}{2} + \frac{(V_2-V_3)}{4} = 0$ (2)

Supernode 3: $\frac{(V_3-V_1)}{8} + \frac{(V_3-V_2)}{4} + \frac{(V_3-0)}{2} + \frac{(V_3-0)}{1} = 0$ (3)

Solving equations 1, 2 and 3, we get:

$\boxed{V_1 = 3.5}$ V,

$\boxed{V_2 = -5.5}$ V, and

$\boxed{V_3 = -0.5}$ V.

∵ source power $P_s = V_s I_s$ (Watt), where I_s is the source current, and V_s is the difference of potential across the source.

∴ $\boxed{P_{2A} = 0}$, and $P_{4A} = (V_1 - V_2) * 4 = [3.5-(-5.5)]$

∴ $\boxed{P_{4A} = 36}$ W