



٦٢٥

(F/h → 6)

الفرقة الأولى

مكناى

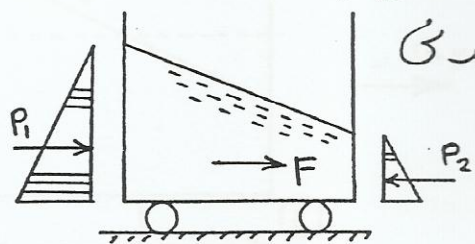
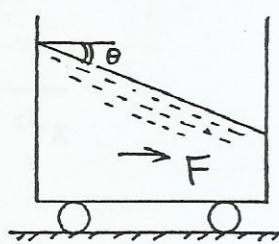
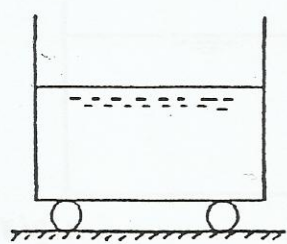
فلويد

A. One

26715909 - 0197000616

Fluid Masses Subjected to linear Acceleration

6

Fluid
سائل

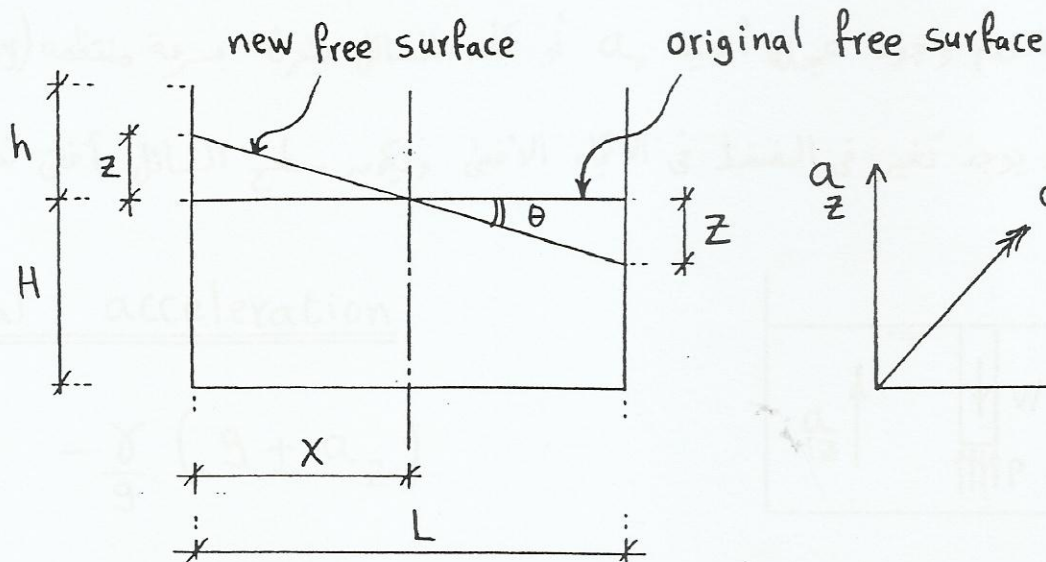
إذا أعطى سائل في إناء مفتوح عجلة منتظمة "a" Uniform acceleration

فإنه بعد زمنه يحرك الإناء كوحدة واحدة moves as a solid body

وبالتالي لا توجد حركة نسبية بين جزيئات السائل وبعضها أو بين الإناء والـ Shear Stresses

في هذه الحالة يمكن تطبيق قوانين الـ Static fluid لكم بإضافة تأثير العجلة

Assume the acceleration (a) in a given direction and its Components a_x , a_z



مكتبة إن - جسي
ب ٧٩ ش البناية عبيد باشا
٤٨٤٢٧٨٦

مكتبة إن - جسي
ب ٧٩ ش البناية عبيد باشا
٤٨٤٢٧٨٦

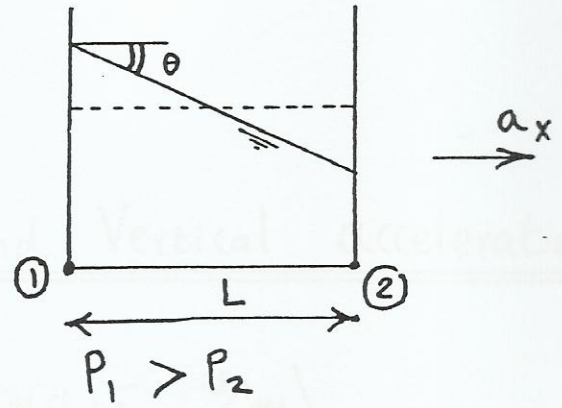
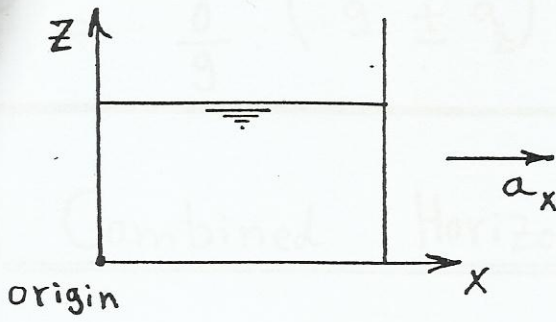
a_x : العجلة في الاتجاه الأفقي

a_z : العجلة في الاتجاه الرأسى

$$\tan \theta = \frac{z}{L/2}$$

الزاوية بين الـ Free surface قبل الحركة والـ Free surface بعد الحركة : θ

(2)

Horizontal acceleration

$$\frac{\partial P}{\partial x} = -\frac{\gamma}{g} a_x$$

$$P_2 = P_1 - \frac{\gamma}{g} a_x L$$

مع هذه المعادلة نستنتج أنه الضغط يتغير في الاتجاه الأفقي وإشارة السالب تعني

أنه الضغط يقل كلما اتجهنا في اتجاه العجلة (إلى اليمين)

السطح تساوي الضغط ليست أفقية لكنه تميل بزاوية θ

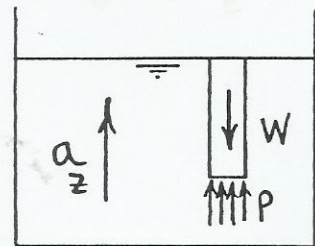
$$* \text{ If } a_x = 0 \Rightarrow \frac{\partial P}{\partial x} = 0$$

في حالة عدم وجود عجله أفقية a_x أو كأنه السائل يتحرك بسرعة منتظمة (Uniform Velocity)

فإنه لا يوجد تغير في الضغط في الاتجاه الأفقي ويكون سطح السائل أفقي تماماً

2 Vertical acceleration

$$\frac{\partial P}{\partial z} = -\frac{\gamma}{g} (g \pm a_z)$$



- إذا تحرك tank محتوياً على سائل رأسياً إلى أعلى بعجلة منتظمة a_z

فإنه الضغط يزداد لحدوثاً مع زيادة العجلة الرأسية إلى أعلى ويظل سطح السائل

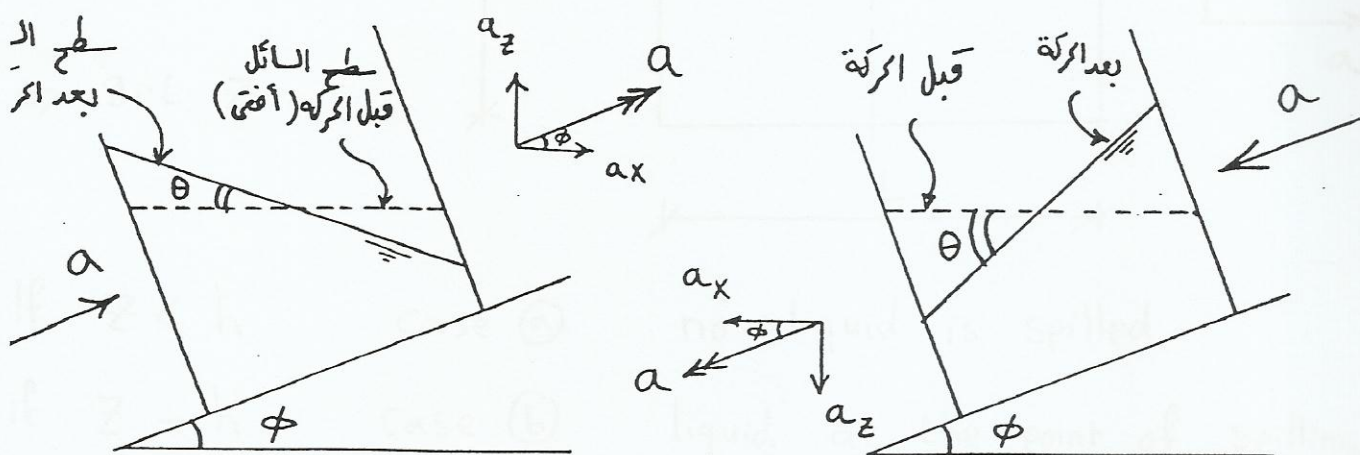
$$* \text{ If } a_z = 0 \Rightarrow \frac{\partial P}{\partial z} = -\gamma$$

أفقياً ما لم يتعرض إلى عجلة أفقية a_x

3

$$P = \frac{\gamma}{g} (g \pm a_z) h$$

3 Combined Horizontal and Vertical acceleration



$$\tan \theta = \frac{a_x}{g \pm a_z}$$

$$a_x = a \cos \phi$$

$$a_z = a \sin \phi$$

لو كان a_z لأعلى $a_z \rightarrow (+ve)$

لو كان a_z لأسفل $a_z \rightarrow (-ve)$

If $a_x = 0 \Rightarrow \tan \theta = 0$

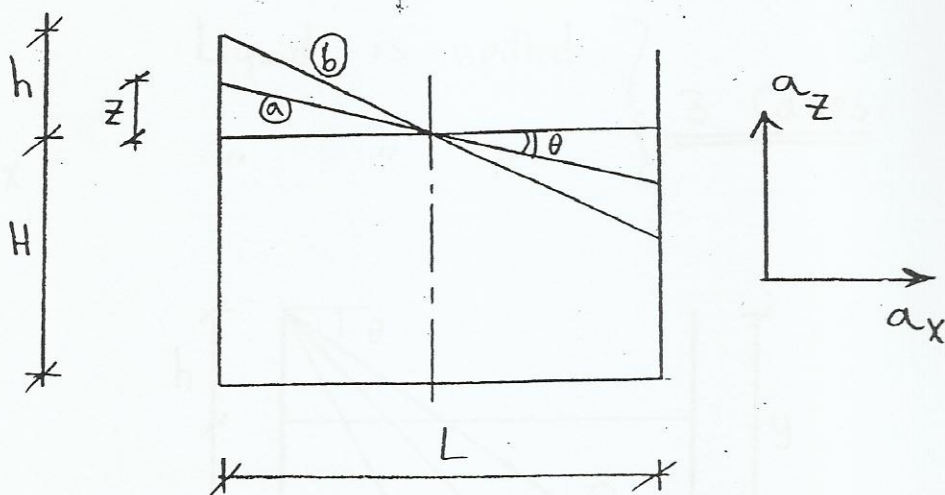
في حالة عدم وجود عجلة أفقية فإن سطح السائل يظل أفقياً

If $a_z = 0 \Rightarrow \tan \theta = \frac{a_x}{g}$

How to know if liquid will be spilt?

$$\tan \theta = \frac{a_x}{g \pm a_z} = \frac{z}{L/2}$$

\Rightarrow get z

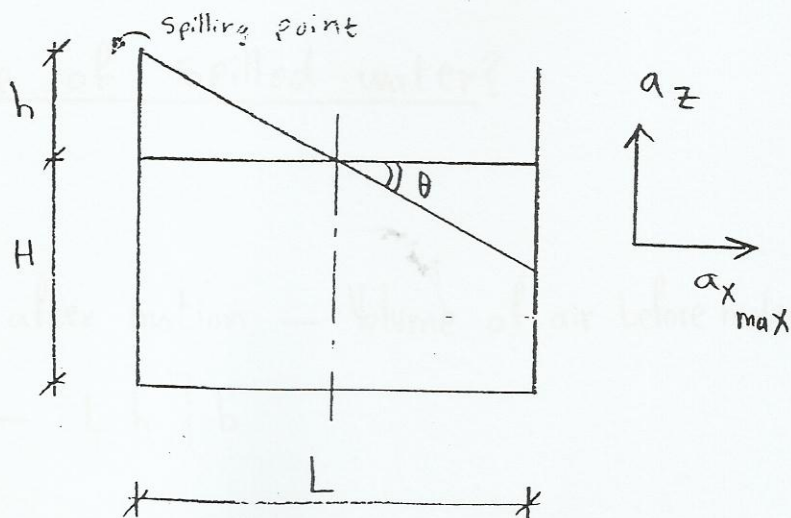


- If $z < h$ case (a) no liquid is spilled
 if $z = h$ case (b) liquid at the point of spilling
 if $z > h$ water is spilled \Rightarrow 3 cases

How to get $(a_x)_{\max}$ or max height of the Container to make the liquid at the spilling point

$$\tan \theta = \frac{a_{x \max}}{g \pm a_z} = \frac{h}{L/2}$$

\Rightarrow get $a_{x \max}$



$$\tan \theta = \frac{a_x}{g \pm a_z} = \frac{h_{\max}}{L/2}$$

\Rightarrow get h_{\max}

(5)

When is liquid spilled?

IF $z > h$

IF $a_x > a_{x_{max}}$

Liquid is spilled } 3 Cases
 " " " }

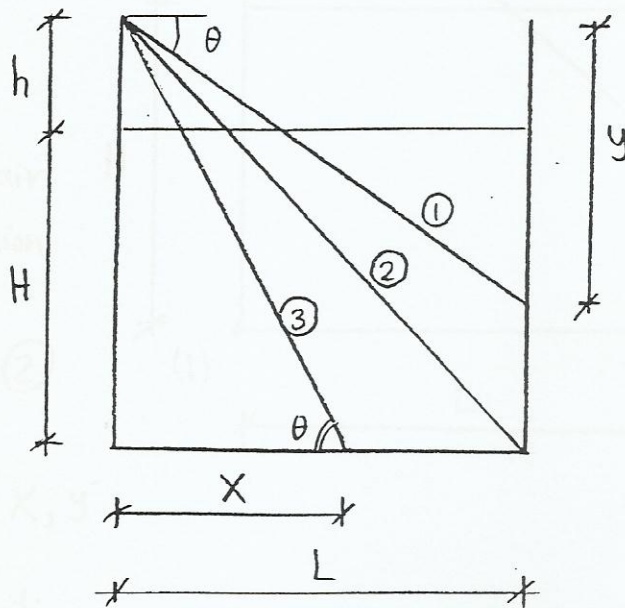
$$\tan \theta = \frac{a_x}{g + a_z} = \frac{y}{L}$$

\Rightarrow get y

IF $y < H + h \Rightarrow \text{Case 1}$

$y = H + h \Rightarrow \text{Case 2}$

$y > H + h \Rightarrow \text{Case 3}$



$$\tan \theta = \frac{a_x}{g + a_z} = \frac{H + h}{X}$$

\Rightarrow get X

How to get the Volume of spilled water?

For case ①, ②

Volume spilled = Volume of air after motion - Volume of air before motion

$$V_{\text{spilled}} = \left[\frac{1}{2} L y - L h \right] b$$

Case ③

Volume spilled = Volume of water before motion - Volume of water after motion

$$V_{\text{spilled}} = \left[L H - \frac{1}{2} X (H + h) \right] b$$

(6)

Case of closed tank, find Pressure at (1) and (2)

$$\tan \theta = \frac{a_x}{g \pm a_z} = \checkmark$$

$$\tan \theta = \frac{y}{X} \rightarrow \textcircled{1}$$

\therefore Area of air before motion = Area of air after motion

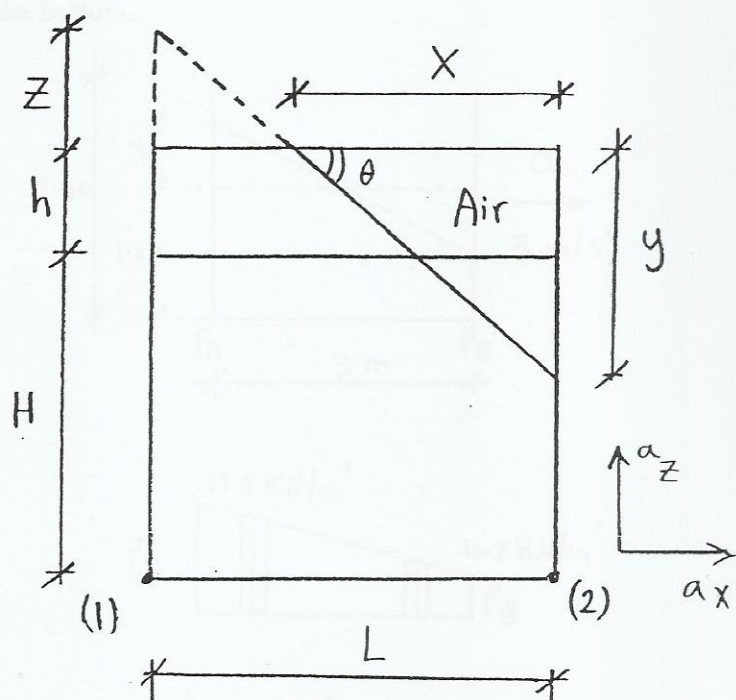
$$L h = \frac{1}{2} X y \rightarrow \textcircled{2}$$

from $\textcircled{1}, \textcircled{2} \Rightarrow$ get X, y

\Rightarrow get Z \leftarrow $\frac{P_1 - P_2}{\rho(g \pm a_z)}$

$$P_1 = P_{\text{air}} + \frac{\gamma}{g} (g \pm a_z) [H + h + Z]$$

$$P_2 = P_{\text{air}} + \frac{\gamma}{g} (g \pm a_z) [H + h - y]$$



- A tank containing water moves horizontally with a constant linear acceleration of 3 m/s^2 .
 The tank is 3 m long, 2.5 m high and the depth of water when the tank is at rest is 1.5 m.
 calculate a) the angle of the water surface to the horizontal,
 b) the maximum pressure intensity on the bottom.

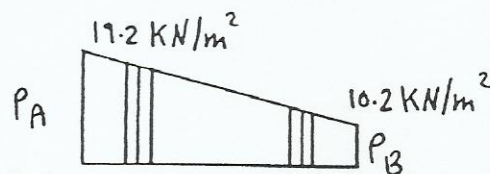
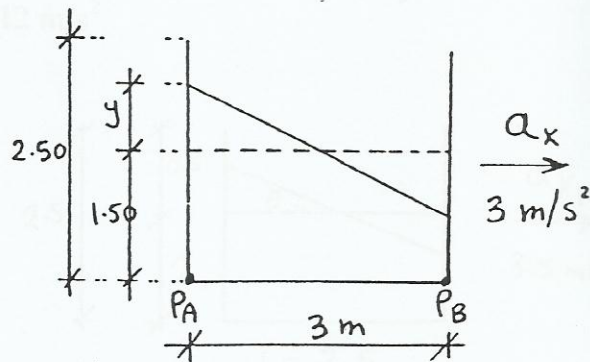
(7)

$$\tan \theta = \frac{a_x}{g}$$

$$\tan \theta = \frac{3}{9.81} \Rightarrow \theta = 17^\circ$$

$$\tan \theta = \frac{3}{9.81} = \frac{y}{x} = \frac{y}{1.5}$$

$$\therefore y = \frac{1.5 \times 3}{9.81} = 0.46$$



$$P_A = \gamma h = 9810(1.5 + 0.46) = 19227 \text{ N/m}^2$$

$$P_B = \gamma h = 9810(1.5 - 0.46) = 10202 \text{ N/m}^2$$

Q) (b) $a_z = 9.806$ $P_a, P_b, P_c = ?$

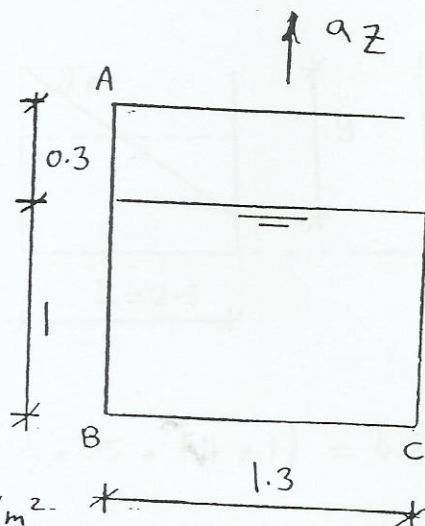
Sol

$$\tan \theta = 0$$

$$P_A = 0$$

$$P_B = P_C = \frac{\gamma}{g}(g \pm a_z)h$$

$$= \frac{9810}{9.81}(9.81 + 9.806)1 = 19616 \text{ N/m}^2$$



If acceleration ceases and the tank moves

at constant velocity

6 m/s

$$\Rightarrow a_z = 0 \Rightarrow P_B = P_C = \gamma h = 9810(1) = 9810 \text{ N/m}^2$$

(8)

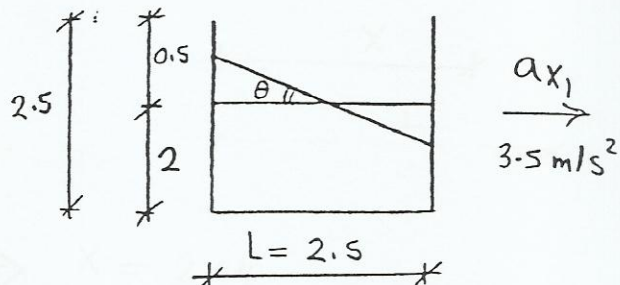
A tank containing water moves horizontally with a constant linear acceleration of 3.5 m/s^2 . The tank is 2.50 m long, 2.5 m high and the depth of water when the tank is at rest is 2 m . Calculate a) the angle of the water surface to the horizontal.

- b) the volume of spilled water when the acceleration is increased by 25%.
c) the force acting on each side if $a = 12 \text{ m/s}^2$.

a) $a_x = 3.5 \text{ m/s}^2$ $\theta = ?$

$$\tan \theta = \frac{a_x}{g} = \frac{3.5}{9.81}$$

$$\therefore \theta = \underline{\underline{19.63^\circ}}$$



b) If a_x is increased 25%

$$a_{x_2} = 1.25 a_{x_1} = 1.25 \times 3.5 = 4.375 \text{ m/s}^2$$

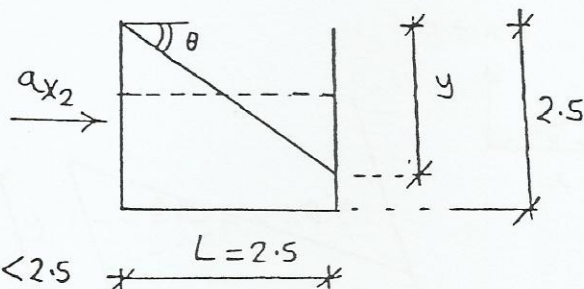
$$\tan \theta = \frac{a_x}{g} = \frac{z}{L/2}$$

$$\frac{4.375}{9.81} = \frac{z}{2.5/2} \Rightarrow z = 0.557 > 0.5 \text{ m}$$

\therefore Water will spill

$$\tan \theta = \frac{a_x}{g} = \frac{y}{L}$$

$$\frac{4.375}{9.81} = \frac{y}{2.5} \Rightarrow y = 1.1 < 2.5$$



$$V_{\text{water after acceleration}} = (2.5 \times 2.5 \times 1) - \left(\frac{1}{2} \times 2.5 \times 1.1 \times 1\right) = 4.86 \text{ m}^3$$

$$V_{\text{water before acceleration}} = (2.5 \times 2 \times 1) = 5 \text{ m}^3$$

$$V_{\text{water spilt}} = V_{\text{water before acceleration}} - V_{\text{after acceleration}} = 5 - 4.86 = \underline{\underline{0.14 \text{ m}^3}}$$

$$\% \text{ Volume spilt} = \frac{0.14}{5} \times 100 = 2.8 \%$$

(9)

$$a_x = 12 \text{ m/s}^2$$

$$\tan \theta = \frac{a_x}{g} = \frac{y}{L}$$

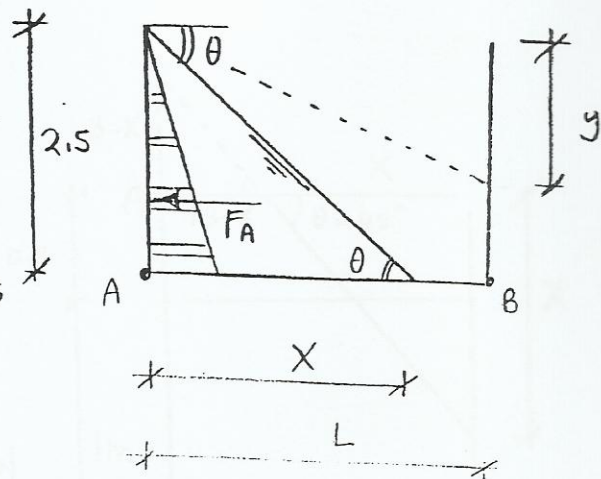
$$\frac{12}{9.81} = \frac{y}{2.5} \Rightarrow y = 3.06 > 2.5$$

$$\therefore \tan \theta = \frac{a_x}{g} = \frac{2.5}{X}$$

$$\frac{12}{9.81} = \frac{2.5}{X} \Rightarrow X = 2.04$$

$$F_A = \gamma A h = 9810 (2.5 \times 1) \frac{2.5}{2} = \underline{\underline{30656 \text{ N}}}$$

$$F_B = 0$$



An open rectangular tank is 5 m long, 4 m wide and 3 m high contains water up to a height of 2 m is accelerated up a 15° inclined plane. Determine the maximum acceleration that can be given without spilling the water along the larger side.

$$\tan(\phi + \theta) = \frac{y}{L/2}$$

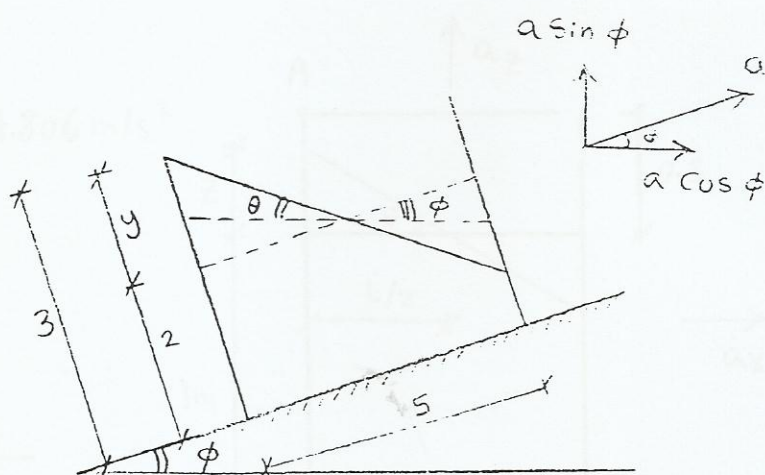
$$\tan(15 + \theta) = \frac{1}{5/2}$$

$$\Rightarrow \theta = 6.8^\circ$$

$$\tan \theta = \frac{a_x}{g + a_z} = \frac{a \cos \phi}{g + a \sin \phi}$$

$$\tan 6.8 = \frac{a \cos 15}{9.81 + a \sin 15}$$

$$\Rightarrow a = \underline{\underline{1.25 \text{ m/s}^2}}$$



① $a_x = 9.806 \text{ m/s}^2$ $P_{a,b,c} = ?$

Sol
 $\tan \theta = \frac{a_x}{g} = \frac{9.806}{9.81} \approx 1$

$\therefore \theta = 45^\circ$

$\therefore V_{\text{air before accel}} = V_{\text{air after accel}}$

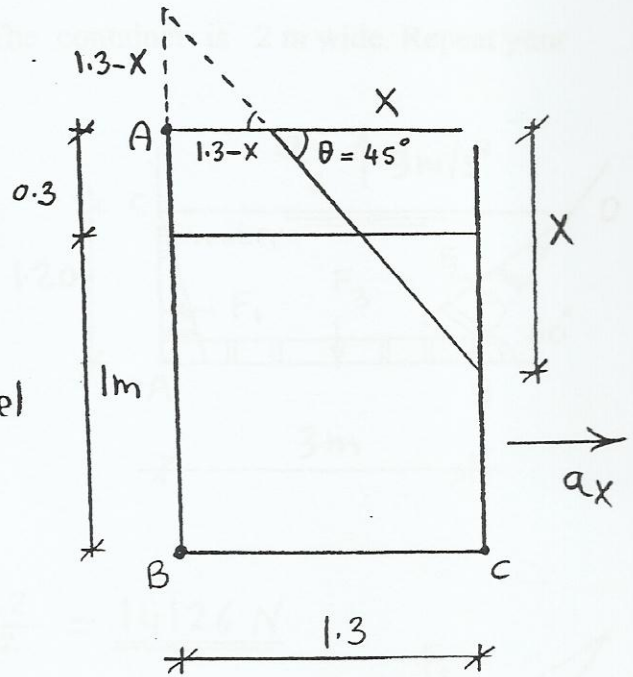
$1.3 \times 0.3 = \frac{1}{2} X^2$

$\therefore X = 0.883$, $1.3 - X = 0.417$

$P_A = \gamma_w (0.417) = \underline{4090 \text{ N/m}^2}$

$P_B = \gamma_w (0.417 + 1.3) = \underline{16843 \text{ N/m}^2}$

$P_C = P_B - \frac{\gamma}{g} a_x L = 16843 - \frac{9810}{9.81} (9.806) 1.3 = \underline{4096 \text{ N/m}^2}$



28 ②

$a_x = 4.903 \text{ m/s}^2$ $a_z = 9.806 \text{ m/s}^2$

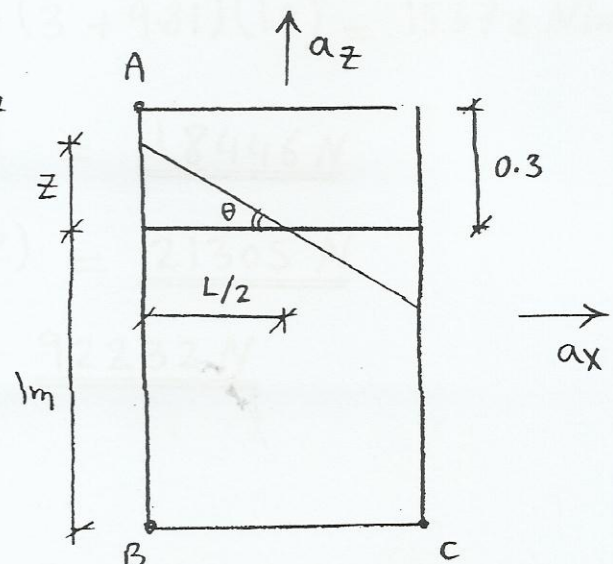
Sol
 $\tan \theta = \frac{a_x}{g + a_z} = \frac{z}{L/2}$
 $= \frac{4.903}{9.81 + 9.806} = \frac{z}{1.3/2}$

$\Rightarrow z = 0.1625 < 0.3 \Rightarrow \underline{P_A = 0}$

$P_B = \frac{\gamma}{g} (g + a_z) h = \frac{9810}{9.81} (9.81 + 9.806) (1 + 0.1625) = \underline{22803 \text{ N/m}^2}$

$P_C = \frac{9810}{9.81} (9.81 + 9.806) (1 - 0.1625) = \underline{16428 \text{ N/m}^2}$

or $P_C = P_B - \frac{\gamma}{g} a_x L = 16428 \text{ N/m}^2$



Calculate the total forces on the ends and bottom of this container while at rest and when being accelerated vertically upward at 3 m/s^2 . The container is 2 m wide. Repeat your calculations for a downward acceleration of 6 m/s^2 .

Width = 2 m

At rest

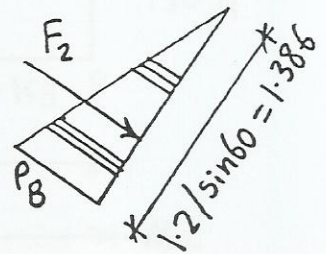
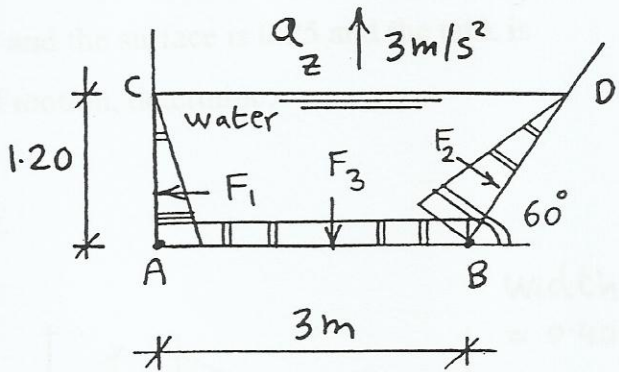
$$P_A = P_B = \gamma h$$

$$= 9810 \times 1.2 = 11772 \text{ N/m}^2$$

$$F_1 = \gamma A \bar{h} = 9810 (1.2 \times 2) \times \frac{1.2}{2} = \underline{14126 \text{ N}}$$

$$F_2 = \frac{1}{2} P_A = \frac{1}{2} (11772) (1.386 \times 2) = \underline{16316 \text{ N}}$$

$$F_3 = P_A = 11772 (3 \times 2) = \underline{70632 \text{ N}}$$



Upward $a_z = 3 \text{ m/s}^2$

$$P_A = P_B = \frac{\gamma}{g} (a_z + g) h = 1000 (3 + 9.81) (1.2) = 15372 \text{ N/m}^2$$

$$F_1 = \frac{1}{2} P_A = \frac{1}{2} (15372) (1.2 \times 2) = \underline{18446 \text{ N}}$$

$$F_2 = \frac{1}{2} P_A = \frac{1}{2} (15372) (1.386 \times 2) = \underline{21305 \text{ N}}$$

$$F_3 = P_A = 15372 \times 3 \times 2 = \underline{92232 \text{ N}}$$

Downward $a_z = 6 \text{ m/s}^2$

$$P_A = P_B = \frac{\gamma}{g} (g - a_z) h = 1000 (9.81 - 6) 1.2 = 4572 \text{ N/m}^2$$

$$F_1 = \frac{1}{2} P_A = \frac{1}{2} (4572) (1.2 \times 2) = \underline{5486 \text{ N}}$$

$$F_2 = \frac{1}{2} P_A = \frac{1}{2} (4572) (1.386 \times 2) = \underline{6336 \text{ N}}$$

$$F_3 = P_A = (4572) \times (3 \times 2) = \underline{27432 \text{ N}}$$

(12)

A rectangular container of base dimensions $0.4 \text{ m} \times 0.2 \text{ m}$ and height 0.5 m is filled with water to a depth of 0.2 m ; the mass of the empty container is 10 kg . The container is placed on a horizontal surface and is subjected to a constant horizontal force of 150 N . If the coefficient of sliding friction between the container and the surface is 0.25 and the tank is aligned with the short dimension along the direction of motion, determine:

- The force of the water on each end of the tank.
- The force of the water on the bottom of the tank.

$$\mu = 0.25$$

$$M_{\text{water}} = 0.2 \times 0.2 \times 0.4 \times 1000 = 16 \text{ Kg}$$

$$M_{\text{full}} = M_{\text{empty}} + M_{\text{water}}$$

$$= 10 + 16 = 26 \text{ Kg}$$

$$F - \mu Mg = Ma$$

$$150 - 0.25(26)9.81 = 26 a_x$$

$$\tan \theta = \frac{a_x}{g} = \frac{3.316}{9.81} = 0.338$$

$$z = \frac{0.2}{2} \tan \theta = 0.1(0.338) = 0.0338 \text{ m}$$

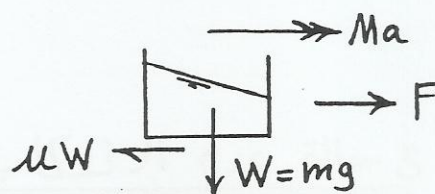
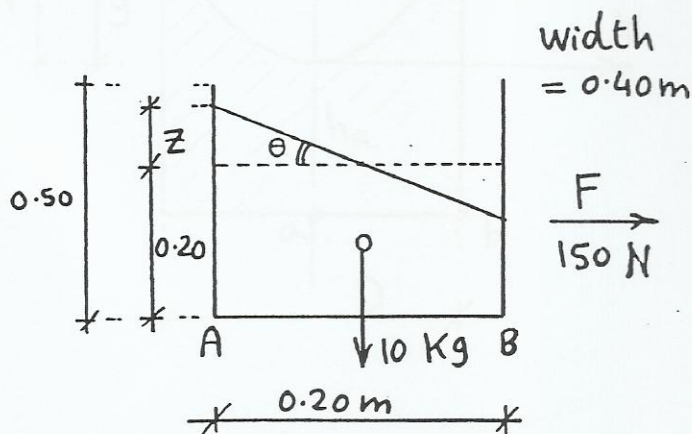
$$F_A = \gamma A \bar{h} = 9810 (0.4 \times 0.2338) \frac{0.2338}{2} = \underline{\underline{107.24 \text{ N}}}$$

$$F_B = \gamma A \bar{h} = 9810 (0.4 \times 0.1662) \frac{0.1662}{2} = \underline{\underline{54.2 \text{ N}}}$$

$$P_A = \gamma h = 9810(0.2338) = 2293.5 \text{ Pa}$$

$$P_B = \gamma h = 9810(0.1662) = 1630.4 \text{ Pa}$$

$$F_{\text{Bottom}} = \frac{1}{2} (P_A + P_B) A = \frac{1}{2} (2293.5 + 1630.4) 0.2 \times 0.4 = \underline{\underline{156.96 \text{ N}}}$$



$$\Rightarrow a_x = 3.316 \text{ m/s}^2$$

Uniform Rotation about Vertical Axis (13)

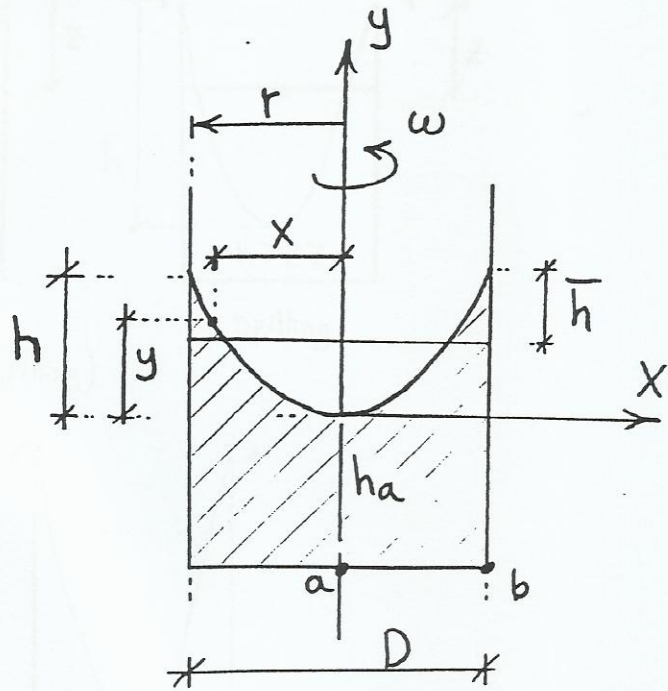
$$\omega = \frac{2\pi N}{60}$$

N = number of revolutions
Per minute (r.p.m.)

ω = Angular Velocity

$$y = \frac{\omega^2 x^2}{2g}$$

$$h = \frac{\omega^2 r^2}{2g}$$



في حالة عدم الانكباب

حجم السائل قبل الدوران = حجم السائل بعد الدوران

حجم الفراغ قبل الدوران = حجم الفراغ بعد الدوران

حجم ال Paraboloid يساوي $\frac{1}{2}$ حجم الاسطوانة المشتركة معه في القاعدة والارتفاع

$$V_{\text{parab}} = \frac{1}{2} \pi r^2 h$$

في حالة عدم الانكباب يقيم ال Paraboloid الى ارتفاعين متساويين

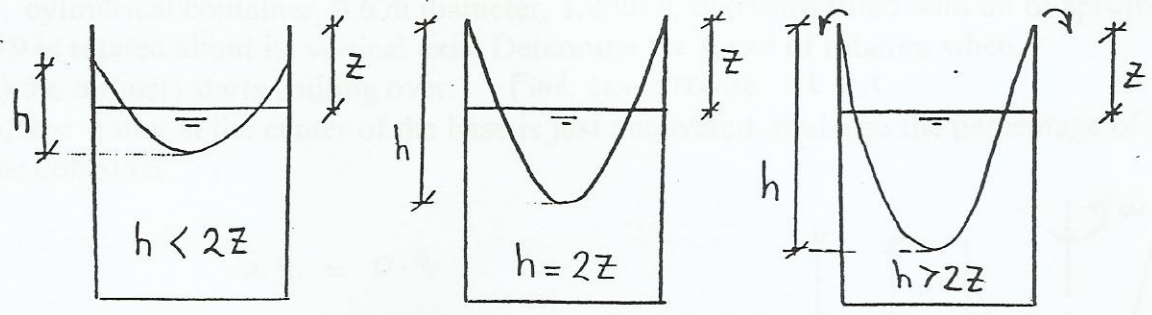
If No liquid is spilled

Volume of air before rotation = Volume of air after rotation

$$\pi r^2 \bar{h} = \frac{1}{2} \pi r^2 h$$

$$\Rightarrow \bar{h} = \frac{h}{2}$$

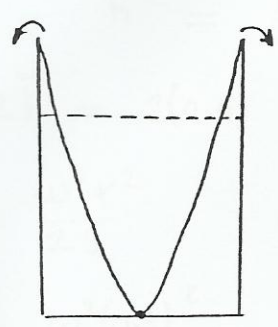
Open Tank Cases



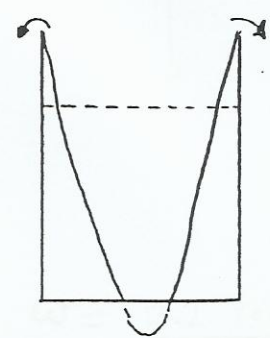
No spilling

No spilling
(at the point of spilling)

spilling

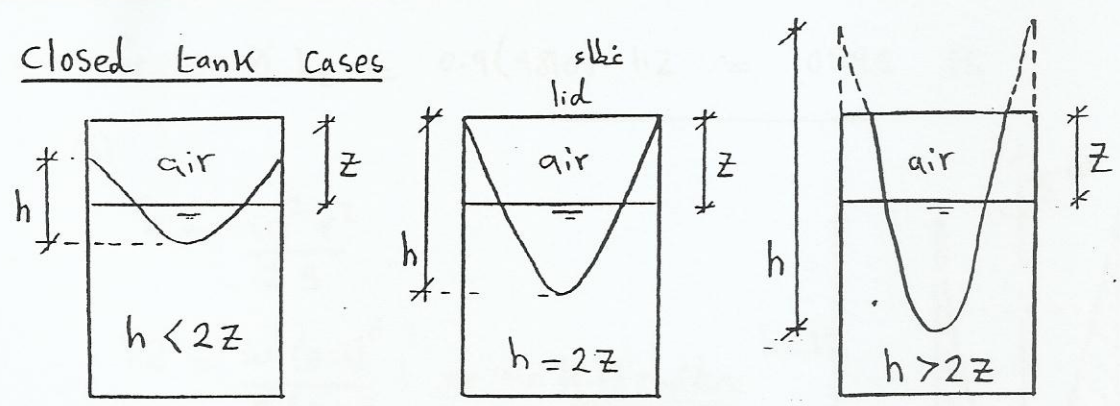


Point at the center
is just uncovered

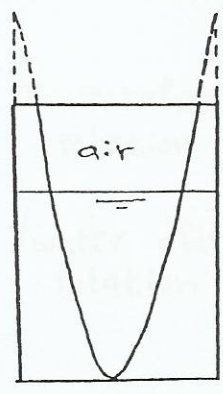


bottom of the tank
is uncovered

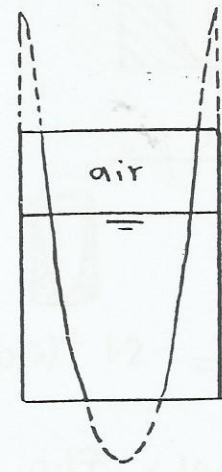
Closed Tank Cases



Water starts to
touch the lid



Point at the center
is just uncovered



bottom of the tank
is uncovered

A cylindrical container, 0.6 m diameter, 1.2 high, two third filled with oil of specific gravity 0.9 is rotated about its vertical axis. Determine the speed of rotation when

- a) the oil just starts spilling over. Find the pressure at B, C
b) the point at the center of the base is just uncovered, find also the percentage of oil left in the container.

$$S.g. = 0.9$$

$$V_{\text{air before rot}} = V_{\text{air after rot}}$$

$$\bar{h} = \frac{h}{2}$$

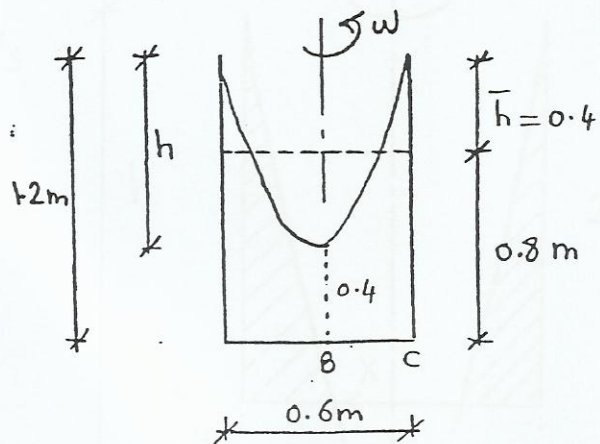
$$h = 2\bar{h} = 2(0.4) = 0.8$$

$$h = \frac{\omega^2 r^2}{2g}$$

$$0.8 = \frac{\omega^2 (0.3)^2}{2 \times 9.81} \Rightarrow \omega = \underline{\underline{13.2 \text{ rad/sec}}}$$

$$P_B = \gamma h_B = 0.9(9810) 0.4 = 3531.6 \text{ Pa}$$

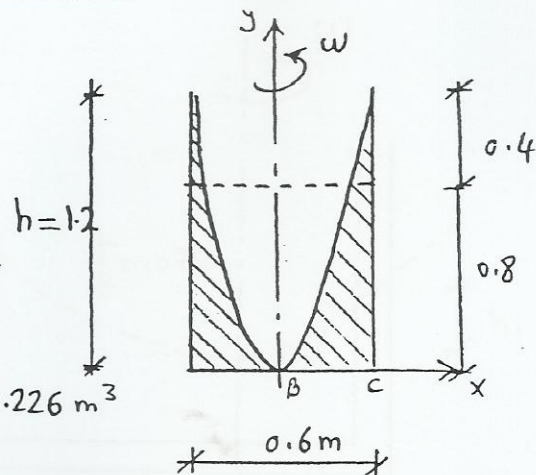
$$P_C = \gamma h_C = 0.9(9810) 1.2 = 10595 \text{ Pa}$$



b)

$$h = \frac{\omega^2 r^2}{2g}$$

$$1.2 = \frac{\omega^2 (0.3)^2}{2(9.81)} \Rightarrow \omega = \underline{\underline{16.17 \text{ rad/sec}}}$$



$$V_{\text{water before rotation}} = \pi r^2 H = \pi (0.3)^2 1.2 = 0.339 \text{ m}^3$$

$$V_{\text{water after rotation}} =$$



$$= \pi (0.3)^2 1.2 - \frac{1}{2} \pi (0.3)^2 1.2 = 0.17 \text{ m}^3$$

$$\% V_{\text{left}} = \frac{V_{\text{after}}}{V_{\text{before}}} \times 100 = \frac{0.17}{0.339} \times 100 = \underline{\underline{75.2 \%}}$$

$$P_B = 0$$

$$P_C = \gamma h_C = 0.9(9810) 1.2 = 10595 \text{ Pa}$$

An open cylindrical tank 2 m high and 1 m in diameter is full of water. If the cylinder is rotated with an angular velocity of 2.5 rev/s.

(16)

How much of the bottom of the tank is uncovered?

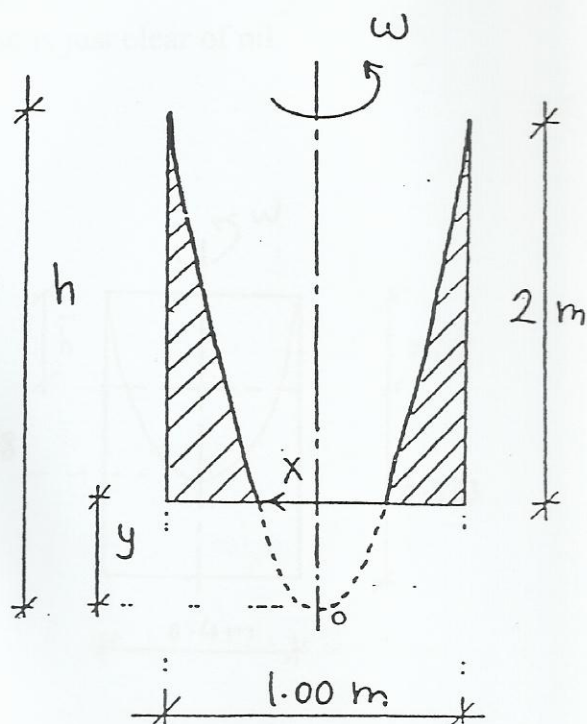
$$\omega = 2\pi n = 2\pi(2.5) = 15.7 \text{ rad/s}$$

$$h = \frac{\omega^2 r^2}{2g}$$

$$= \frac{(15.7)^2 (0.5)^2}{2 \times 9.81} = 3.14$$

$$y = h - 2 = 3.14 - 2 = 1.14 \text{ m}$$

$$1.14 = \frac{(15.7)^2 x^2}{2 \times 9.81} \Rightarrow x = 0.30 \text{ m}$$



$$\text{Uncovered Area} = \pi x^2 = \pi (0.3)^2 = \underline{\underline{0.285 \text{ m}^2}}$$

29

$$h_2 - h_1 = 0.075$$

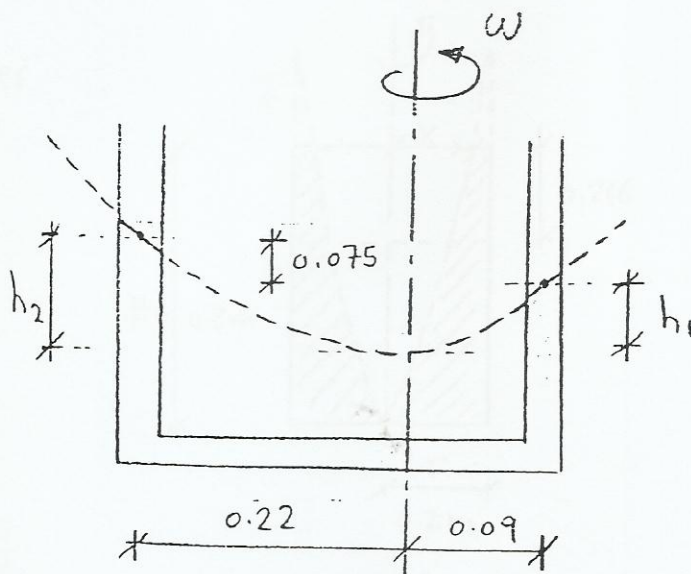
$$\omega = ?$$

$$h_1 = \frac{\omega^2 r^2}{2g} = \frac{\omega^2 (0.09)^2}{2(9.81)}$$

$$h_2 = \frac{\omega^2 (0.22)^2}{2(9.81)}$$

$$\frac{\omega^2 (0.22)^2}{2(9.81)} - \frac{\omega^2 (0.09)^2}{2(9.81)} = 0.075$$

$$\Rightarrow \omega = \underline{\underline{6.04 \text{ rad/s}}}$$



A closed cylindrical container, 0.40 m diameter and 0.8 m high, two third of its height is filled with oil (S.G. = 0.85). The container is rotated about its vertical axis. Determine the speed of rotation when: a) the oil just start touching the lid.

b) the point at the center of the base is just clear of oil.

$$S.G. = 0.85$$

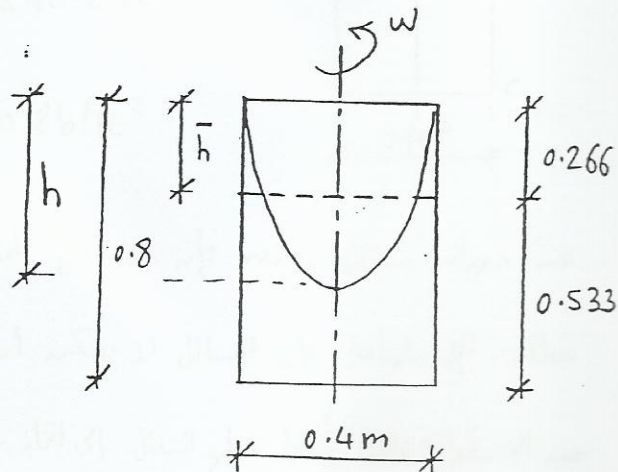
$$V_{\text{air before}} = V_{\text{air after}}$$

$$\bar{h} = \frac{h}{2}$$

$$h = 2\bar{h} = 2(0.266) = 0.533$$

$$h = \frac{\omega^2 r^2}{2g}$$

$$0.533 = \frac{\omega^2 (0.2)^2}{2 \times 9.81} \Rightarrow \omega = \underline{\underline{16.17 \text{ rad/sec}}}$$



$$V_{\text{air before}} = V_{\text{air after}}$$

$$\pi r^2 (0.266) = \frac{1}{2} \pi x^2 (0.8)$$

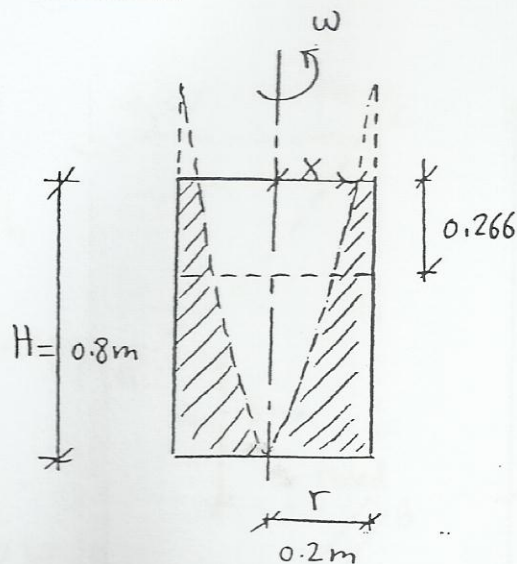
$$(0.2)^2 (0.266) = \frac{1}{2} x^2 0.8$$

$$\therefore x = 0.163 \text{ m}$$

$$H = \frac{\omega^2 x^2}{2g}$$

$$0.8 = \frac{\omega^2 (0.163)^2}{2 \times 9.81}$$

$$\Rightarrow \omega = \underline{\underline{24.29 \text{ rad/sec}}}$$



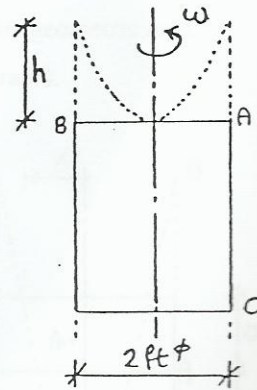
18 A closed vessel, 2 ft in diameter, is completely filled with water. If the vessel is rotated at 1200 rpm, what increase in pressure would occur at the top of the tank at the circumference?

18

$$\omega = \frac{2\pi N}{60} = \frac{2\pi (1200)}{60} = 125.6 \text{ rad/s}$$

$$h = \frac{\omega^2 r^2}{2g} = \frac{(125.6)^2 (1)^2}{2 \times 32.2} = 245.2 \text{ ft}$$

$$P = \gamma h = 62.4 (245.2) = 15300 \text{ lb/ft}^2$$



عند دوران سائل مغلقة فيه الأسطح العليا عند A, B تصبح تحت ضغط ، ونظراً لوجود غطاء للإسطوانة فيه السائل لا يمكنه أن يأخذ شكل Parabola لأنه الغطاء يعلقه عليه المزج من الأسطوانة ، لذا يأخذ سطح السائل إلى انماذ شكل Parabola وتصبح النقطة A تحت ضغط $\gamma \frac{\omega^2 r^2}{2g}$

A tube AOB has the part OB, 300 mm long, bent upwards so that the angle OB is 45°. AO is vertical and the end B is closed. The tube AOB is completely filled with water to a height of 230 mm above O. Find the number of revolutions per min of the tube about axis OA so that the pressure at B is the same as the pressure at O.

$$P_O = P_B$$

$$\gamma h_{AO} = \gamma h_{BC}$$

$$\therefore BC = AO = 230 \text{ mm}$$

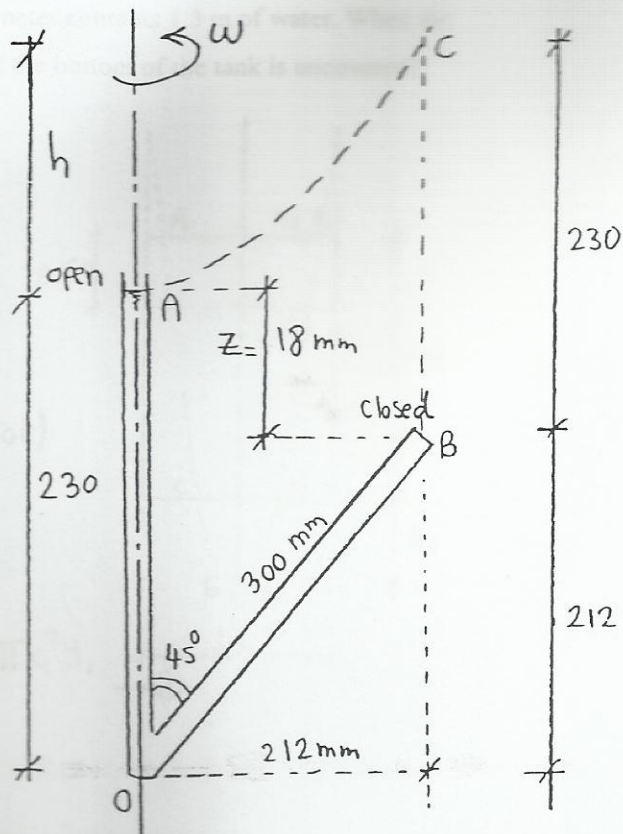
$$Z = 230 - 300 \cos 45^\circ = 18 \text{ mm}$$

$$h = \frac{\omega^2 r^2}{2g}$$

$$(0.23 - 0.018) = \frac{\omega^2 (0.212)^2}{2 \times 9.81}$$

$$\Rightarrow \omega = 9.62 \text{ rad/s} = \frac{2\pi N}{60}$$

$$\therefore N = \underline{\underline{91.86 \text{ rev/min}}}$$



A closed cylindrical tank, 2 m high and 1 m in diameter, contains 1.5 m of water with air space subjected to a pressure of 1.07 bar. If the cylinder rotates about its geometric axis. What are the pressures at points C and D, when the angular velocity is 12 rad/s.

$$H = \frac{\omega^2 r^2}{2g} = \frac{(12)^2 (0.5)^2}{2 \times 9.81} = 1.83 \text{ m}$$

$$V_{\text{air}} (\text{before rot}) = V_{\text{air}} (\text{after rot})$$

$$\pi (0.5)^2 \cdot 0.5 = \frac{1}{2} \pi x^2 y$$

$$x^2 y = 0.25 \rightarrow \textcircled{1}$$

$$y = \frac{(12)^2 x^2}{2 \times 9.81} \rightarrow \textcircled{2}$$

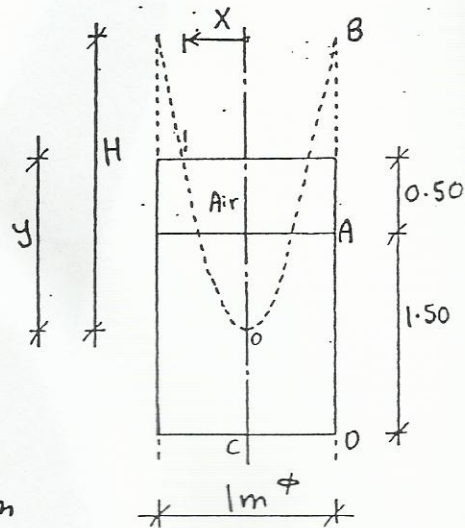
$$y^2 = \frac{(12)^2 \cdot 0.25}{2 \times 9.81} = 1.83 \Rightarrow y = 1.36 \text{ m}$$

$$C = 2 - 1.36 = 0.64 \text{ m}$$

$$BD = 0.64 + 1.83 = 2.47$$

$$= 1.07 \times 10^5 + 0.64 \times 9810 = \underline{\underline{1.13 \text{ bar}}}$$

$$P_D = 9810(2.47) + 1.07 \times 10^5 = \underline{\underline{1.31 \text{ bar}}}$$



A closed cylindrical tank 2 m high and 1 m in diameter contains 1.5 m of water. When the angular velocity is constant at 20 rad/s, how much of the bottom of the tank is uncovered?

$$y_1 = \frac{(20)^2 x_1^2}{2 \times 9.81} \rightarrow \textcircled{1}$$

$$y_2 = \frac{(20)^2 x_2^2}{2 \times 9.81} \rightarrow \textcircled{2}$$

$$V_{\text{air}} (\text{before rot}) = V_{\text{air}} (\text{after rot})$$

$$\frac{\pi (1)^2}{4} \times 0.5 = \frac{\pi x_2^2 (2 + y_1)}{2} - \frac{\pi x_1^2 y_1}{2}$$

$$0.3927 = \frac{1}{2} \pi x_2^2 (2 + y_1) - \frac{1}{2} \pi x_1^2 y_1 \rightarrow \textcircled{3}$$

$$\text{from } \textcircled{1}, \textcircled{2}, \textcircled{3} \Rightarrow x_1 = 0.116$$

$$\text{Area} = \pi (0.116)^2 = \underline{\underline{0.0423 \text{ m}^2}}$$

