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الانتقال الحراري في طبقة مسامية أفقية تقع تحت طبقة مائع أفقية تحتوي على مجال مغناطيسي

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ملخص البحث :

في هذا البحث تمت دراسة حالة الاستقرار بالصورة الخطية لنموذج يتكون من طبقة مائع أفقية تتأثر بمجال مغناطيسي عمودي وتعتلى طبقة أفقية مسامية تتعرض للتسخين من الأسفل . وقد افترض في هذا البحث أن معادلة دارسي تتحكم في المائع الموجود في الطبقة المسامية . وقد تمت دراسة حالة الاستقرار الثابت لهذا النموذج على افتراض تطبيق شرط بيفرز وجوزيف بين الطبقتين . وتم الحصول على حلول عددية لهذا النموذج باستخدام طريقة كثيرة حدود شيبيشف ووجد أن هذا النوع من الطرق العددية قادر على حل المسائل ذات الطبقات المتعددة كما وجد أن وجود المجال المغناطيسي له تأثير كبير على هذا النموذج .

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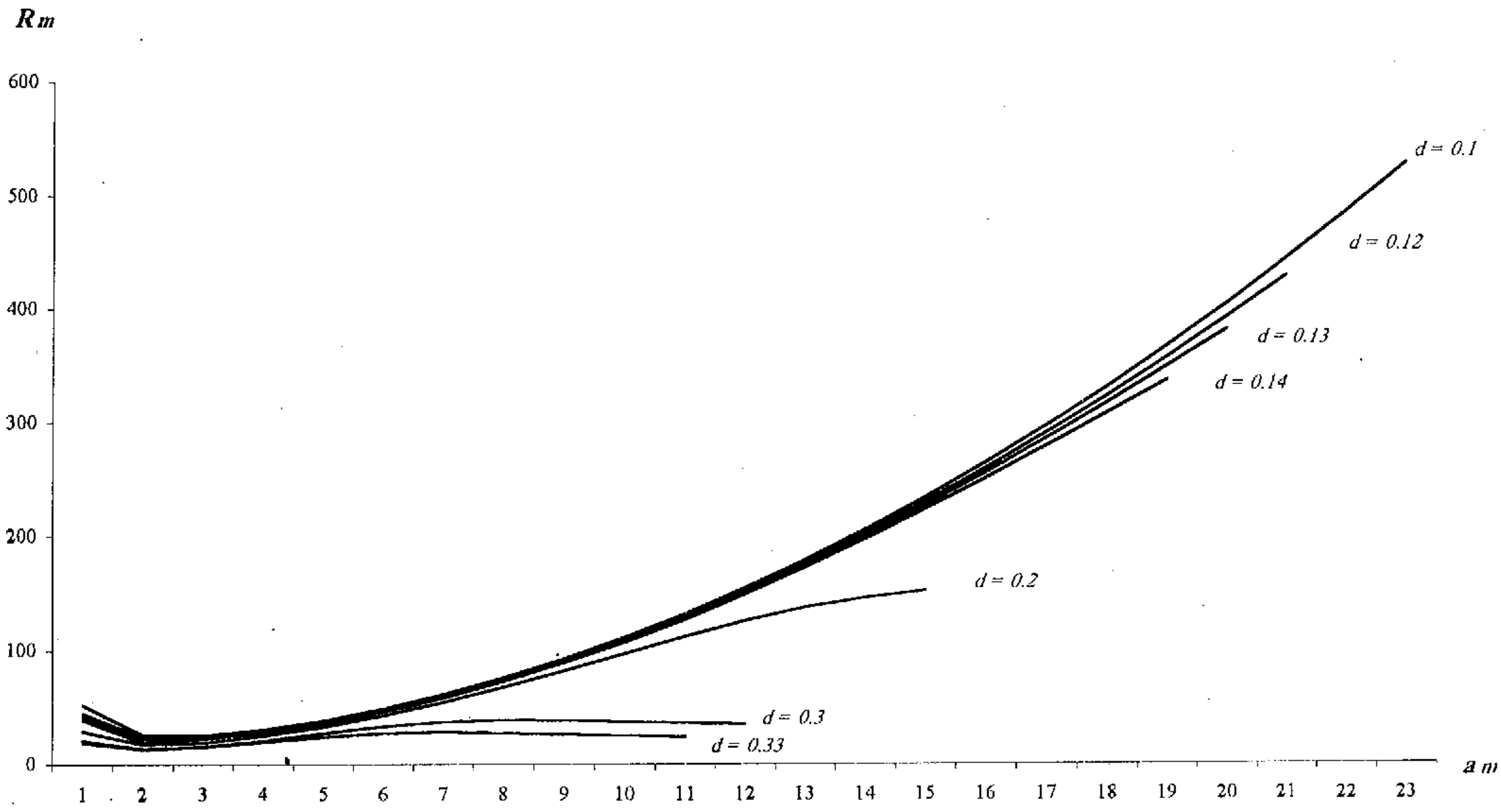
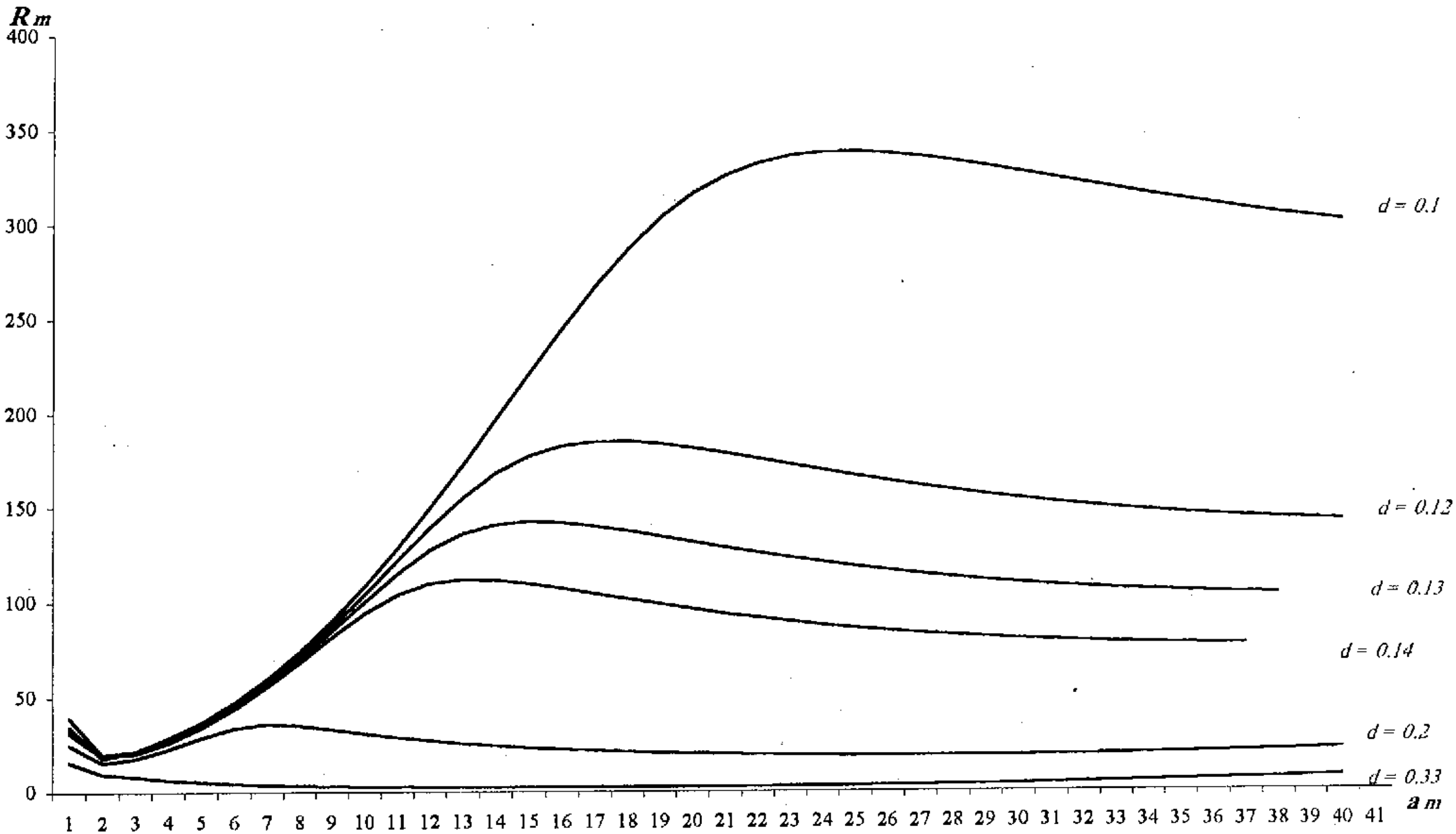
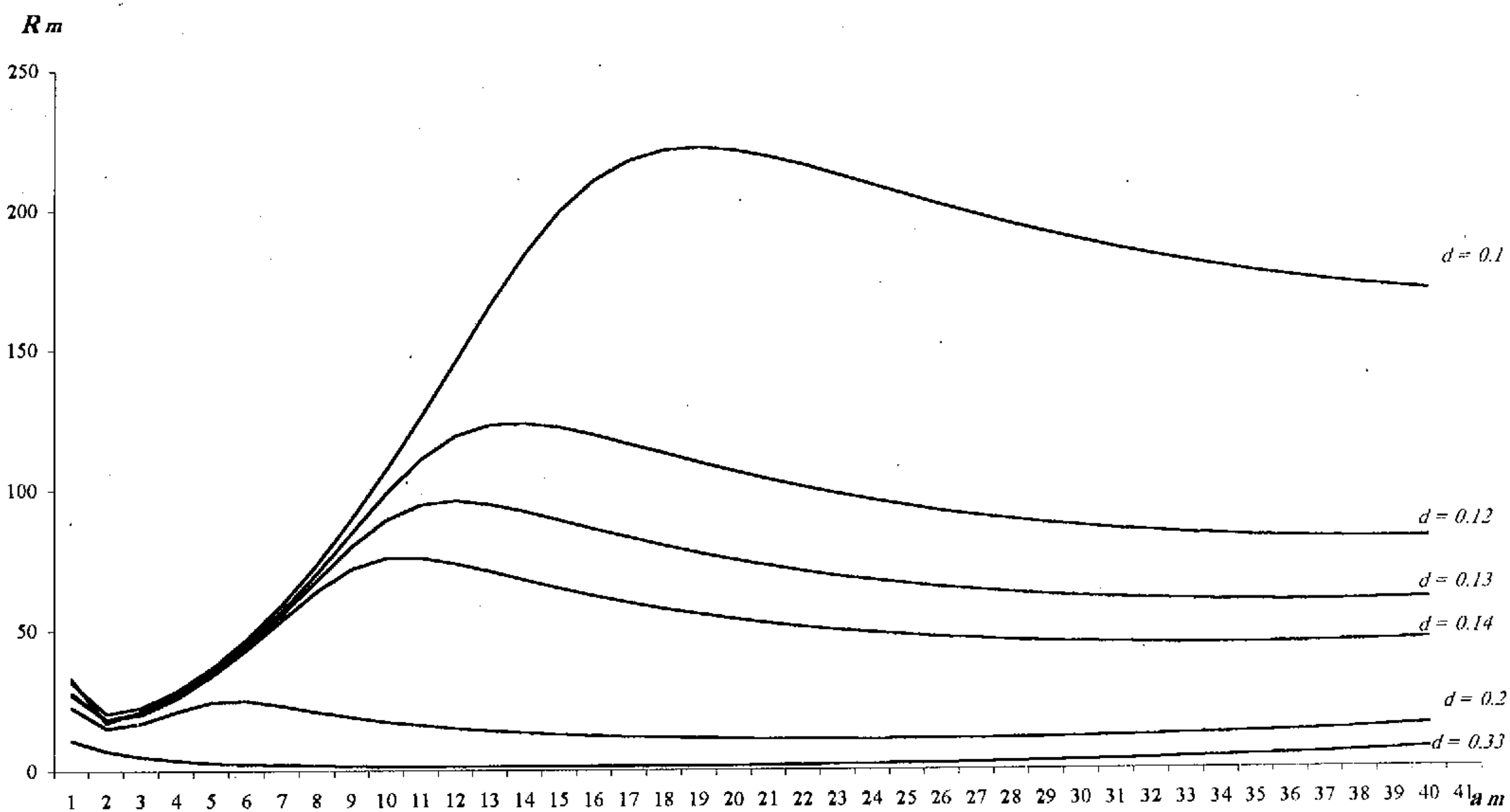


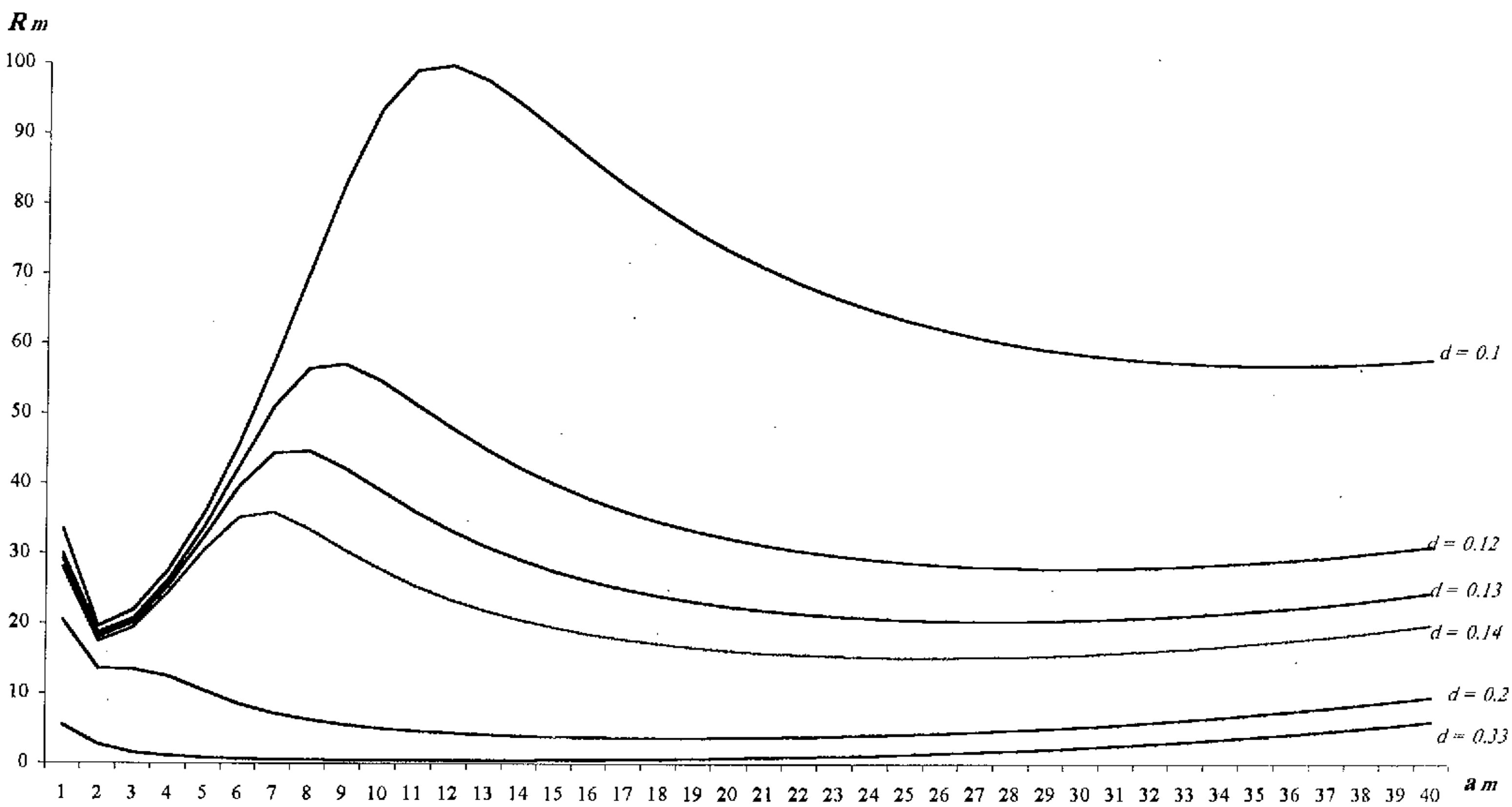
Figure(4): The relation between a_m and R_m when $Q = 10000$.



Figure(3): The relation between R_m and a_m when $Q = 1000$.



Figure(2): The relation between R_m and a_m when $Q = 500$.



Figure(1): The relation between a_m and R_m when $Q = 100$.

Figure (1) shows the relation between a_m and R_m for different values of the depth ratio of $\hat{d}$ when the Chandrasekhar number $Q = 100$. It is clear from the figure that the Rayleigh number in the porous layer decreases continuously as the thickness of the layer increases. The results correspond to $Q = 500, 1000, 10000$ are displayed in figures (2) - (4) respectively. It is clear from these figures that the Rayleigh number increases as the Chandrasekhar number increases, i.e. the magnetic field has a stabilizing effect on the system .

differential equation in the porous layer with 12 boundary conditions. This problem is solved using spectral method based on series expansion of Chebyshev polynomials. In producing the results σ_f and σ_m are set to zero identically which corresponds to the stationary convection instability where the relation between σ_f and σ_m are given by

$$\sigma_f = \frac{\hat{d}^2}{\hat{k}}\sigma_m .$$

Numerical results and stability curves are obtained for the problem, with thermal conductivity ratio $\hat{\kappa} = 1.43$, Darcy number $= 4 \times 10^{-6}$, Beavers-Joseph constant $\alpha_{BJ} = 0.1$ and for a variety of reciprocal depth ratios ranging from 0.33 to 0.1. The results of this paper are illustrated in figures (1) - (4). They are qualitatively and quantitively similar to those produced by Bukhari [10] in the absence of magnetic field in the fluid layer. Bukhari has showed that the numerical results produced by Chen and Chen [4] have a large rounding error due to the method used which is a 4th order Runge-Kutta method and he showed that the spectral methods have a strong ability to solve the multi-layered problems and produce accurate results.

$$G_m \sigma_m \theta_m = w_m + \left(D_m^2 - a_m^2\right)\theta_m ,$$

where $a_m^2 = r_m^2 + q_m^2$, $a_f^2 = r_f^2 + q_f^2$ are non-dimensional wave numbers in the porous medium and fluid layers respectively and where $D(\ ) = \dfrac{\partial}{\partial x_3}(\ )$, $a_f = \hat{d}\, a_m$, $\sigma_f = \dfrac{\hat{d}^2}{\hat{\kappa}}\sigma_m$. The final boundary conditions are :

<u>Upper boundary $x_3 = 1$</u>

$$w_f = 0 , \quad D_f w_f = 0 , \quad \theta_f = 0 , \quad h_f = 0 . \tag{2.37}$$

<u>Middle boundary $x_3 = 0$</u>

$$\theta_f = \varepsilon_T \theta_m , \quad D_f \theta_f = D_m \theta_m , \quad w_m = \varepsilon_T w_f , \quad D_f h_f = 0 ,$$

$$\varepsilon_T \hat{d} \left(D_f w_f - \frac{\hat{d}\sqrt{Da}}{\alpha_{BJ}} D_f^2 w_f \right) = D_m w_m , \tag{2.38}$$

$$\hat{d}^3 \varepsilon_T Da \left(D_f^3 w_f - 3a_f^2 D_f w_f - \frac{\sigma_f}{P_{r_f}} D_f w_f \right) = -\left(\frac{Da}{\varphi} \frac{\sigma_m}{P_{r_m}} + 1 \right) D_m w_m .$$

<u>Lower boundary $x_3 = -1$</u>

$$w_m = 0 , \quad \theta_m = 0 \tag{2.39}$$

3. Results and Discussion

The eigenvalue problem consists of an eighth order ordinary differential equation in the fluid layer and a fourth order ordinary

velocity. It can be shown that these boundary conditions are transformed to

$$\hat{d}^3 \varepsilon_T Da \frac{\partial}{\partial x_3}\left(\nabla^2 w_f - \frac{1}{P_{r_f}} \frac{\partial w_f}{\partial t} + 2\nabla^2 w_f \right) = -\left(1 + \frac{Da}{\varphi P_{r_m}} \frac{\partial}{\partial t} \right) \frac{\partial w_m}{\partial x_3}$$

(2.35)

$$\varepsilon_T \hat{d} \frac{\partial}{\partial x_3}\left(w_f - \frac{\hat{d}\sqrt{Da}}{\alpha_{BJ}} \frac{\partial w_f}{\partial x_3} \right) = \frac{\partial w_m}{\partial x_3} .$$

If we consider that

$$w_m(t, \boldsymbol{x}) = w_m(x_3) \, exp[\, i(r_m x_1 + q_m x_2) + \sigma_m t \,],$$

$$\theta_m(t, \boldsymbol{x}) = \theta_m(x_3) \, exp[\, i(r_m x_1 + q_m x_2) + \sigma_m t \,],$$

$$w_f(t, \boldsymbol{x}) = w_f(x_3) \, exp[\, i(r_f x_1 + q_f x_2) + \sigma_f t \,],$$

$$\theta_f(t, \boldsymbol{x}) = \theta_f(x_3) \, exp[\, i(r_f x_1 + q_f x_2) + \sigma_f t \,],$$

$$h_f(t, \boldsymbol{x}) = h_f(x_3) \, exp[\, i(r_f x_1 + q_f x_2) + \sigma_f t \,]$$

where r_m , r_f , q_m and q_f are wavenumbers while σ_m and σ_f are the growth rates. It follows from equations (2.33) and (2.34) that

$$\frac{\sigma_f}{P_{r_f}}\left(D_f^2 - a_f^2\right) w_f - \sigma Q P_{m_f}^{-1} D h_f = \left(D_f^2 - a_f^2\right)^2 w_f - Q D^2 w_f - Ra_f a_f^2 \theta_f$$

$$\sigma_f \theta_f = w_f + \left(D_f^2 - a_f^2\right)\theta_f ,$$

$$\sigma_f P_{m_f}^{-1} h_f = \left(D_f^2 - a_f^2\right) h_f + D w_f , \qquad (2.36)$$

$$-\frac{Da}{\varphi} \frac{\sigma_m}{P_{r_m}} \left(D_m^2 - a_m^2\right) w_m = \left(D_m^2 - a_m^2\right) w_m + Ra_m a_m^2 \theta_m ,$$

$$\frac{Da}{\varphi}\frac{\partial \boldsymbol{V}_m}{\partial t_m} = P_{r_m}\left[-\nabla_m p_m - \boldsymbol{V}_m + Ra_m \theta_m \boldsymbol{e}_3\right]$$

$$G_m \frac{\partial \theta_m}{\partial t_m} - \beta w_m = \nabla_m^2 \theta_m \qquad (2.32)$$

where $\beta = sign\,(T_o - T_u) = sign\,(T_\ell - T_o)$ and the hat superscript has been dropped although the variables are non-dimensional. By taking the *curl curl* of the momentum equation in each layer then taking the third component of the equations in each layer, equations (2.31) and (2.32) become

$$\frac{\partial}{\partial t}\left(\frac{1}{P_{r_f}}\nabla^2 w_f - Q P_{m_f}^{-1}\frac{\partial \boldsymbol{h}_f}{\partial z}\right) = \nabla^4 w_f - Q\frac{\partial^2 \boldsymbol{w}_f}{\partial z^2} + Ra_f \nabla^2 \theta_f$$

$$\frac{\partial \theta_f}{\partial t} - \beta w_f = \nabla^2 \theta_f \qquad (2.33)$$

$$P_{m_f}^{-1}\frac{\partial \boldsymbol{h}_f}{\partial t} = \frac{\partial w_f}{\partial z} + \nabla_f^2 \boldsymbol{h}_f$$

and

$$\frac{1}{P_{r_m}}\frac{Da}{\varphi}\frac{\partial}{\partial t}\nabla^2 w_m = -\nabla^2 w_m + Ra_m \nabla^2 \theta_m$$

$$G_m \frac{\partial \theta_m}{\partial t} - \beta w_m = \nabla^2 \theta_m \, . \qquad (2.34)$$

The Beavers-Joseph and normal stress interfacial boundary conditions must be reworked to eleminate pressure and horizontal components of

The scaling (2.24) and (2.27) are now used to non-dimensionalize the boundary conditions (2.23). Thus

$$\hat{\theta}_f(1) = 0, \quad \hat{w}_f(1) = 0, \quad \frac{\partial \hat{w}_f}{\partial x_3} = 0, \quad \hat{h}_f(1) = 0,$$

$$\hat{\theta}_f(0) = \varepsilon_T\, \hat{\theta}_m(0), \quad \frac{\partial \hat{\theta}_f(0)}{\partial x_3} = \frac{\partial \hat{\theta}_m(0)}{\partial x_3}, \quad \frac{\partial h_f}{\partial x_3} = 0,$$

$$\varepsilon_T\, \hat{d}\, Da\left(\hat{p}_f - 2\frac{\partial \hat{w}_f}{\partial x_3}\right) = \hat{p}_m, \quad \varepsilon_T\, \hat{w}_f(0) = \hat{w}_m(0), \quad (2.30)$$

$$\varepsilon_T \frac{\partial \hat{u}_f}{\partial x_3} = \frac{\alpha_{BJ}}{\hat{d}\sqrt{Da}}\left(\varepsilon_T\, \hat{u}_f - \hat{u}_m\right), \; \varepsilon_T \frac{\partial \hat{v}_f}{\partial x_3} = \frac{\alpha_{BJ}}{\hat{d}\sqrt{Da}}\left(\varepsilon_T\, \hat{v}_f - \hat{v}_m\right)$$

$$\hat{\theta}_m(-1) = 0, \quad \hat{w}_m(-1) = 0,$$

where the parameters ε_T, $\hat{d}$ and $\hat{\kappa}$ are defined by

$$\varepsilon_T = \frac{\hat{d}}{\hat{\kappa}}, \qquad \hat{d} = \frac{d_m}{d_f}, \qquad \hat{\kappa} = \frac{\kappa_m}{\kappa_f}.$$

The linearized version of equations (2.25) and (2.28) can be obtained by ignoring all product term. For the fluid layer $\hat{\boldsymbol{V}}_f$, θ_f and $\boldsymbol{h}_f$ satisfy

$$\frac{\partial \boldsymbol{V}_f}{\partial t} = P_{r_f}\left[-\nabla_f\, p_f + \nabla_f^2 \boldsymbol{V}_f + Ra_f\,\theta_f\, \boldsymbol{e}_3 + Q_f \frac{\partial \boldsymbol{h}_f}{\partial z}\right]$$

$$\frac{\partial \theta_f}{\partial t_f} - \beta w_f = \nabla_f^2\, \theta_f \qquad (2.31)$$

$$P_{m_f}^{-1} \frac{\partial \boldsymbol{h}_f}{\partial t} = \frac{\partial \boldsymbol{V}_f}{\partial z} + \nabla_f^2\, \boldsymbol{h}_f$$

and for the porous layer $\boldsymbol{V}_m$ and θ_m satisfy

A similar procedure is applied to the porous medium layer in which non-dimensional spatial coordinates $\hat{x}_m$, time $\hat{t}_m$, perturbed velocity $\hat{V}_m$, pressure $\hat{p}_m$ and temperature $\hat{\theta}_m$ are introduced by the definitions

$$x_m = d_m \hat{x}_m , \qquad t_m = \frac{d_m^2}{\lambda_m}\hat{t}_m , \qquad \hat{V}_m = \frac{\lambda_m}{d_m}\hat{V}_m$$

$$p_m = \frac{\mu \lambda_m}{K}\hat{p}_m ; \qquad \theta_m = | \ T_o - T_u \ | \hat{\theta}_m \tag{2.27}$$

where λ_m is the thermal diffusivity of the porous medium layer defined by $\lambda_m = \frac{K_m}{(\rho c_p)_f}$. Then equations (2.22) governing the motion of the fluid in the porous layer become

$$\frac{Da}{\varphi}\frac{\partial \hat{V}_m}{\partial \hat{t}_m} = P_{r_m}\left[-\nabla_m \hat{p}_m - \hat{V}_m + Ra_m \hat{\theta}_m e_3\right],$$

$$G_m \frac{\partial \hat{\theta}_m}{\partial \hat{t}_m} + \hat{V}_m \cdot \left(\nabla_m \hat{\theta}_m - sign(T_\ell - T_o)\ e_3\right) = \nabla_m^2 \hat{\theta}_m , \tag{2.28}$$

where $G_m = \frac{(\rho c)_m}{(\rho c_p)_f}$ while P_{r_m} , Da and Ra_m denote, respectively the Prandtle number, Darcy number and Rayleigh number of the porous layer and are defined by

$$P_{r_m} = \frac{\nu}{\lambda_m} , \quad Da = \frac{K}{d_m^2} , \quad Ra_m = \frac{g\alpha K d_m |T_1 - T_o|}{\nu \lambda_m} . \tag{2.29}$$

$$x = d_f \hat{x}_f , \qquad t_f = \frac{d_f^2}{\lambda_f} \hat{t}_f , \qquad V_f = \frac{\lambda_f}{d_f} \hat{V}_f$$

$$p_f = \frac{\mu \lambda_f}{d_f^2} \hat{p}_f , \qquad h_f = \frac{H_f \lambda_f}{\eta_f} \hat{h}_f , \qquad \theta_f = | T_o - T_u | \hat{\theta}_f \tag{2.24}$$

where λ_f is the thermal diffusivity of the fluid phase defined by $\lambda_f = \frac{\kappa_f}{(\rho c_p)_f}$. Equations (2.21) describing the motion of the fluid layer may be written in the non-dimensional form

$$\frac{\partial \hat{V}_f}{\partial \hat{t}_f} + (\hat{V}_f . \nabla_f) \hat{V}_f = P_{r_f} \left[-\nabla_f \hat{p}_f + \nabla_f^2 \hat{V}_f + Ra_f \hat{\theta}_f e_3 + Q \frac{\partial \hat{h}_f}{\partial z} + P_{m_f}^{-1} \hat{h}_f . \nabla_f \hat{h}_f \right]$$

$$\frac{\partial \hat{\theta}_f}{\partial \hat{t}_f} + \hat{V}_f . \left(\nabla_f \hat{\theta}_f - sign(T_o - T_u) \ e_3 \right) = \nabla_f^2 \hat{\theta}_f \tag{2.25}$$

$$P_{m_f}^{-1} \left(\frac{\partial \hat{h}_f}{\partial \hat{t}_f} + (\hat{V}_f . \nabla_f) \hat{h}_f - \hat{h}_f . \nabla_f \hat{V}_f \right) = \frac{\partial \hat{V}_f}{\partial z} + \eta_f \nabla_f^2 \hat{h}_f$$

where P_{r_f} is the viscous Prandtle number , P_{m_f} the magnetic Prandtle number , Q the Chandrasekhar number and Ra_f the Rayleigh number are defined by

$$P_{r_f} = \frac{\nu}{\lambda_f} , \quad P_{m_f} = \frac{\eta_f}{\lambda_f} , \quad Q = \frac{\mu H_f^2 d_f^2}{4 \rho_o \pi \nu \eta_f} , \quad Ra_f = \frac{g \alpha d_f^3 |T_o - T_u|}{\nu \lambda_f} . \tag{2.26}$$

$$\frac{\rho_0}{\varphi}\frac{\partial V_m}{\partial t} = -\nabla p_m - \frac{\mu}{k}V_m - \rho_0 \alpha\, \theta_m\, g \,,$$

$$(\rho\, c)_m \frac{\partial\, \theta_m}{\partial t} + (\rho\, c_p)_f V_m \cdot \left(\nabla \theta_m - \frac{T_l - T_o}{d_m}\, e_3 \right) = \kappa_m \nabla^2 \theta_m \,. \tag{2.22}$$

The modified boundary conditions on the upper boundary of the fluid layer ($x_3 = d_f$), the fluid / porous interface ($x_3 = 0$) and the porous layer ($x_3 = -d_m$) are respectively

$$\theta_f\,(d_f) = 0, \quad w_f\,(d_f) = 0, \quad \frac{\partial w_f(d_f)}{\partial x_3} = 0, \quad h_f(d_f) = 0,$$

$$\theta_f\,(0) = \theta_m\,(0), \quad \kappa_f \frac{\partial \theta_f(0)}{\partial x_3} = \kappa_m \frac{\partial \theta_m(0)}{\partial x_3},$$

$$-p_f(0) + 2\mu \frac{\partial w_f(0)}{\partial x_3} = -p_m(0), \quad w_f\,(0) = w_m\,(0), \tag{2.23}$$

$$\frac{\partial u_f(0)}{\partial x_3} = \frac{\alpha_{BJ}}{\sqrt{K}}\big(u_f(0) - u_m(0)\big), \quad \frac{\partial \upsilon_f(0)}{\partial x_3} = \frac{\alpha_{BJ}}{\sqrt{K}}\big(\upsilon_f(0) - \upsilon_m(0)\big)$$

$$\theta_m\,(-d_m) = 0, \quad w_m\,(-d_m) = 0.$$

The condition (2.14) become $\dfrac{\partial h_f(0)}{\partial x_3} = 0$ where h_f is the third component of the perturbed magnetic field. In the fluid layer we shall introduce the non-dimensional spatial coordinates $\hat{x}_f$; time $\hat{t}_f$, perturbed velocity $\hat{V}_f$, pressure $\hat{p}_f$, magnetic field $\hat{h}_f$ and temperature $\hat{\theta}_f$ by the definitions

$$T_f = T_o - (T_o - T_u)\frac{x_3}{d_f} \qquad 0 \le x_3 \le d_f \quad .$$

$$(2.19)$$

$$T_m = T_o - (T_\ell - T_o)\frac{x_3}{d_m} \qquad -d_m \le x_3 \le 0 \; .$$

Suppose small perturbation for the static state so that the velocity, pressure, temperature and magnetic field in the fluid and porous layers are respectively

$$\boldsymbol{V}_f \;, \quad P_f + p_f \;, \quad T_o - (T_o - T_u)\frac{x_3}{d_f} + \theta_f \;, \quad H\,\delta_{i3} + \boldsymbol{h}_f$$

and (2.20)

$$\boldsymbol{V}_m \;, \quad P_m + p_m \;, \quad T_o - (T_\ell - T_o)\frac{x_3}{d_m} + \theta_m \; .$$

It follows from the general field equations (2.9) and (2.10) that $\boldsymbol{V}_f$, p_f , θ_f and $\boldsymbol{h}_f$ satisfy

$$\rho_0\left(\frac{\partial \boldsymbol{V}_f}{\partial t} + (\boldsymbol{V}_f.\nabla \boldsymbol{V}_f)\right) =$$

$$-\nabla p_f + \mu\nabla^2 \boldsymbol{V}_f - \rho_0\,\alpha\,\theta_f\,\boldsymbol{g} + \frac{\mu}{4\pi}\left(H\frac{\partial \boldsymbol{h}_f}{\partial z} + \boldsymbol{h}_f\cdot\nabla \boldsymbol{h}_f\right)$$

$$\left(\rho\, \mathrm{c_p}\right)_f\left[\frac{\partial\,\theta_f}{\partial t} + \boldsymbol{V}_f\cdot\left(\nabla\theta_f - \frac{T_0 - T_u}{d_f}\,\boldsymbol{e}_3\right)\right] = \kappa_f\,\nabla^2\,\theta_f \qquad (2.21)$$

$$\frac{\partial \boldsymbol{h}_f}{\partial t} + (\boldsymbol{V}_f.\nabla)\,\boldsymbol{h}_f = (\boldsymbol{h}_f.\nabla)\,\boldsymbol{V}_f + H\frac{\partial \boldsymbol{V}_f}{\partial z} + \eta_f\nabla^2\boldsymbol{h}_f \;,$$

whereas $\boldsymbol{V}_m$, p_m and θ_m satisfy .

$$\frac{\partial u_f(0)}{\partial x_3} = \frac{\alpha_{BJ}(u_f - u_m)}{\sqrt{K}}, \qquad \frac{\partial v_f(0)}{\partial x_3} = \frac{\alpha_{BJ}(v_f - v_m)}{\sqrt{K}} \qquad (2.15)$$

where u_f , v_f are the limiting tangential components of the fluid velocity as the interface is approached from the fluid layer $\mathcal{C}'_1$, whereas u_m , v_m are the same limiting components of tangential fluid velocity as the interface is approached from the porous layer $\mathcal{C}'_2$.

Equations (2.9) and (2.10) posses a static (equilibrium) solution satisfying the boundary conditions (2.11) - (2.15) of the form

$$\boldsymbol{V}_f = \boldsymbol{0}, \qquad \boldsymbol{V}_m = \boldsymbol{0}, \qquad \boldsymbol{H}_f = (0, 0, H)$$

$$(2.16)$$

$$-\nabla P_m + \rho_f \boldsymbol{g} = \boldsymbol{0}, \quad -\nabla P_f + \rho_f \boldsymbol{g} = \boldsymbol{0}, \quad \nabla^2 T_m = \nabla^2 T_f = 0$$

together with the exterior boundary conditions

$$T_f\,(d_f) = T_u\,, \qquad T_m\,(-d_m) = T_l \qquad (2.17)$$

and the interfacial conditions

$$T_m(0) = T_f(0), \quad \kappa_m \frac{\partial T_m(0)}{\partial x_3} = \kappa_f \frac{\partial T_f(0)}{\partial x_3}, \qquad P_f(0) = P_m(0).$$

$$(2.18)$$

In conclusion, it follows that the equilibrium temperature fields in the fluid and porous media are respectively

be impenetrable and at constant temperature T_1. In terms of w_f and w_m, the axial velocity components of the fluid in $\boldsymbol{C}'_1$ and $\boldsymbol{C}'_2$ respectively, these requirements leads to the three conditions

$$T_f\,(d_f) = T_u\ ,\ w_f\,(d_f) = 0\ ,\ \frac{\partial w_f(d_f)}{\partial x_3} = 0\ ,\ H\,(d_f) = 0 \quad (2.11)$$

on the top boundary of $\boldsymbol{C}'_1$ and the two conditions

$$T_m\,(-d_m) = T_1 \quad ,\quad w_m\,(-d_m) = 0 \qquad (2.12)$$

on the lower boundary of $\boldsymbol{C}'_2$ where H is the third component of the magnetic field vector. The fluid / porous - media interface boundary conditions are based on the assumption that temperature, heat flux and normal fluid velocity are continuous. Thus at $x_3 = 0$ we have

$$\begin{aligned} &T_m\,(0) = T_f\,(0), && \kappa_m \frac{\partial T_m(0)}{\partial x_3} = \kappa_f \frac{\partial T_f(0)}{\partial x_3} \\ &w_m\,(0) = w_f\,(0), && -P_f(0) + 2\mu \frac{\partial w_f(0)}{\partial x_3} = -P_m(0)\,. \end{aligned} \qquad (2.13)$$

This leaves two final conditions to be specified on the interface. One of these is related to the magnetic field which is

$$\frac{\partial H(0)}{\partial x_3} = 0 \qquad (2.14)$$

and the final one is due to Beavers and Joseph [9] which has the form

pressure P_f. The governing equations for porous medium layer are represented by (see Bukhari [10])

$$\frac{\rho_o}{\varphi}\frac{\partial V_m}{\partial t} = -\nabla P_m - \frac{\mu}{k}V_m + \rho_f g ,$$

$$(\rho c)_m \frac{\partial T_m}{\partial t} + (\rho c_p)_f V_m \cdot \nabla T_m = \kappa_m \nabla^2 T_m , \tag{2.10}$$

where T_m is the Kelvin temperature of the porous medium layer, V_m is the solenoidal seepage velocity, P_m is the hydrostatic pressure, k is the permeability of the porous medium, φ is its porosity, κ_m is the overall thermal conductivity of the porous medium layer and $(\rho c)_m$ is the overall heat capacity per unit volume of porous medium layer at constant pressure. In fact

$$(\rho c)_m = \varphi(\rho c_p)_f + (1 - \varphi)(\rho c_p)_m$$

where $(\rho c_p)_m$ is the heat capacity per unit volume of the porous substrate.

The convection problem is completed by the specification of boundary conditions at the upper surface of the viscous fluid layer, at the interface between the fluid and porous medium layers and at the lower boundary of the porous medium layer. For comparison with Chen and Chen [4] we shall assume that $x_3 = d_f$ is rigid and held at constant temperature T_u, whereas $x_3 = -d_m$ is assumed to

$$\frac{\partial \boldsymbol{H}_f}{\partial t} = (\boldsymbol{H}_f \cdot \nabla)\boldsymbol{V}_f - (\boldsymbol{V}_f \cdot \nabla)\boldsymbol{H}_f + \eta_f \nabla^2 \boldsymbol{H}_f \qquad (2.7)$$

The relations (2.4), $(2.5)_2$ and (2.3) are used to recast the Lorentz force $\boldsymbol{J} \times \boldsymbol{B}$ into

$$\boldsymbol{J}_f \times \boldsymbol{B}_f = \frac{\mu_f}{4\pi}(curl\ \boldsymbol{H}_f) \times \boldsymbol{H}_f = \frac{\mu_f}{4\pi}\left[(\boldsymbol{H}_f \cdot \nabla)H_f - \nabla\left(\frac{\boldsymbol{H}_f^2}{2}\right)\right] \qquad (2.8)$$

The field equations for this problem are written separately for the superposed fluid layer and porous medium layer. The governing equations for the fluid layer are

$$\rho_0\left(\frac{\partial \boldsymbol{V}_f}{\partial t} + (\boldsymbol{V}_f \cdot \nabla)\boldsymbol{V}_f\right) = -\nabla P_f + \mu\nabla^2 \boldsymbol{V}_f + \rho_f g + \frac{\mu_f}{4\pi}\boldsymbol{H}_f \cdot (\nabla \boldsymbol{H}_f)$$

$$(\rho c_p)_f\left(\frac{\partial T_f}{\partial t} + \boldsymbol{V}_f \cdot \nabla T_f\right) = \kappa_f \nabla^2 T_f , \qquad (2.9)$$

$$\frac{\partial \boldsymbol{H}_f}{\partial t} = (\boldsymbol{H}_f \cdot \nabla)\boldsymbol{V}_f - (\boldsymbol{V}_f \cdot \nabla)\boldsymbol{H}_f + \eta_f \nabla^2 \boldsymbol{H}_f .$$

Here T_f is the Kelvin temperature of the fluid layer, P_f is the fluid kinetic pressure, $\boldsymbol{g}$ is the acceleration due to gravity, μ is the dynamic viscosity coefficient of the fluid, $(\rho c_p)_f$ is the heat capacity per unit volume of the fluid at constant pressure and κ_f is the thermal conductivity of the fluid. The electromagnetic pressure $\frac{\mu_f}{4\pi}\left[-\nabla\left(\frac{\boldsymbol{H}_f^2}{2}\right)\right]$ is neglected in this problem and has been added to the form of the kinetic

$$div\ \boldsymbol{B}_f = 0 \tag{2.3}$$

Suppose that the magnetization in the fluid is directly proportional to the applied field and that the fluid behaves like an Ohmic conductor so that $\boldsymbol{H}_f$, $\boldsymbol{B}_f$, $\boldsymbol{J}_f$ and $\boldsymbol{E}_f$ are connected by the relations

$$\boldsymbol{B}_f = \mu_f \boldsymbol{H}_f \ , \qquad \boldsymbol{J}_f = \overline{\sigma}\left(\boldsymbol{E}_f + \boldsymbol{V}_f \times \boldsymbol{B}_f\right) \tag{2.4}$$

and the Maxwell equations

$$curl\ \boldsymbol{E}_f = -\frac{\partial \boldsymbol{B}_f}{\partial t} \qquad (2.5)_1$$

$$\boldsymbol{J}_f = \frac{1}{4\pi}\ curl\ \boldsymbol{H}_f \qquad (2.5)_2$$

where μ_f (constant) is the magnetic permeability, $\overline{\sigma}$ is the electrical conductivity and the displacement current has been neglected in the second of these Maxwell equations as is customary in situation when free charge is instantaneously dispersed. On taking the curl of equations $(2.5)_2$ and replacing the electric field by the Maxwell relation $(2.5)_1$, the magnetic field $\boldsymbol{H}_f$ is now readily seen to satisfy the partial differential equation

$$\eta_f\ curl\ curl\ \boldsymbol{H}_f = -\frac{\partial \boldsymbol{H}_f}{\partial t} + curl\ (\ \boldsymbol{V}_f \times \boldsymbol{H}_f\ ) \tag{2.6}$$

where $\eta_f = \left(4\pi \mu_f \overline{\sigma}\right)^{-1}$ is the electrical resistivity. Equation (2.6) is now reworked using standard vector identities to yield

The fluid flow in the porous layer $\mathcal{C}_2$, with thickness d_m, is governed by Darcy's law, whereas the fluid flow in the upper layer $\mathcal{C}_1$ with thickness d_f, is governed by Navier-Stokes equations. Convection is driven by the temperature dependence of the fluid density. Typically, the Oberbeck-Boussinesq approximation is made in which concepts like local thermal equilibrium, heating from viscous dissipation, radiation effects etc. are ignored as are variations in fluid density except where they occurs only in the momentum equation. Let T denotes the Kelvin temperature of the fluid and T_o be a constant reference Kelvin temperature then for the purpose of this work, the fluid density ρ_f is related to T by

$$\rho_f = \rho_o\left[1 - \alpha(T - T_o)\right] \tag{2.1}$$

where ρ_o is the density of the fluid at T_o and α the coefficient of volume expansion of the fluid. (α is supposed constant)

Let $\boldsymbol{V}_f$, $\boldsymbol{H}_f$, $\boldsymbol{B}_f$, $\boldsymbol{J}_f$ and $\boldsymbol{E}_f$ be respectively the fluid layer velocity, magnetic field, magnetic induction, current density and electric field. The incompressibility of the fluid and the non-existence of magnetic monopoles require that $\boldsymbol{V}_f$ and $\boldsymbol{B}_f$ are both solenoidal vectors. Hence

$$div\ \boldsymbol{V}_f = 0 \tag{2.2}$$

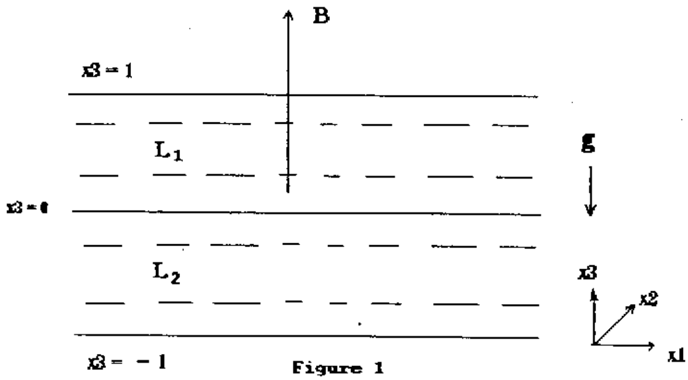


Figure 1

Suppose that the upper layer $\mathcal{L}_1$ is filled with an incompressible thermally and electrically conducting viscous fluid and is subjected to electromagnetic force whereas the lower layer $\mathcal{L}_2$ is occupied by a porous medium permeated by the fluid. Gravity acts in the negative x_3 direction and the porous medium is heated at its lower boundary such that the temperature of the lower boundary is greater than that of the fluid and the porous media. Convection takes place in which temperature driven buoyancy effects are damped by viscous effects. A stationary fluid with a thermal gradient in the x_3 direction (the so called " conduction solution") is one possible solution to this problem and so it is natural to investigate its stability.

superposed by a fluid layer affected by a vertical magnetic field. The flow in the porous layer is assumed to be governed by Darcy's law. The linear stability equations will be solved using expansion of Chebyshev polynomials. This method has been used by Abdullah [6] in the study of the Benard problem in the presence of a non-linear magnetic fluid and by Lindsay and Ogden [7] in the implementation of spectral methods resistant to the generation of spurious eigenvalues. Lamb [8] used also this method to investigate an eigenvalue problem arising from a model discussing the instability in the earth's core. The method possesses good convergence characteristics and effectively exhibits exponential convergence rather than finite power convergence.

2. Mathematical formulation

Let $\mathcal{C}'_1$ and $\mathcal{C}'_2$ be two horizontal layers such that the bottom of the layer $\mathcal{C}'_1$ touches the top of the layer $\mathcal{C}'_2$. A right handed system of Cartesian coordinates (x_i , i = 1,2,3) is chosen so that the interface is the plane $x_3 = 0$, the top boundary of $\mathcal{C}'_1$ is $x_3 = d_f$ which is rigid and the lower boundary of $\mathcal{C}'_2$ is $x_3 = -d_m$. (figure 1)

that the critical Rayleigh number in the porous layer decreases continuously as the thickness of the fluid layer is increased and he used a shooting method to solve the linear stability equations. Nield [2] formulated the problem with surface-tension effects at a deformable upper surface included and he obtained asymptotic solutions for small wave numbers for a constant heat-flux boundary condition. Sun [1] and Nield [2] have used Darcy's law in formulating the equations for porous layer while Somerton and Catton [3] used the Brinkman term in the equation of motion to solve the problem using Galerkin method. Recently, Chen and Chen [4] considered the problem with temperature and salinity gradients are exist in both layers. Their investigation assumed stationary instability from the outset and they used a shooting technique based on fourth order Runge-Kutta approximations for integration of all differential equations. Chen et. al [5] studied the problem with an isotropic permeability and thermal diffusivity in the porous layer. Flow in porous layer was assumed to be governed by Darcy's law and the linearized stability equations are solved using shooting method.

In the present study, we shall emulate the work of Chen and Chen [4] in the presence of a vertical magnetic field. i.e. we shall consider the onset of thermal convection in a horizontal porous layer

Convection in a Horizontal Porous Layer Superposed by a Fluid layer in the Presence of Magnetic Field

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Abstract

A linear stability analysis is applied to a system consisting of a horizontal fluid layer, affected by a vertical magnetic field, superposed a layer of a porous medium permeated by the fluid with uniform heating from below. Flow in porous medium is assumed to be governed by Darcy's law. The Beavers-Joseph condition is applied at the interface between the two layers. Numerical solutions were obtained for stationary convection case using the method of expansion of Chebyshev polynomials. It is found that the spectral method has a strong ability to solve the multi-layered problem and that the magnetic field has a strong effect in this model .

1. Introduction

The onset of convection in a system consisting of a horizontal fluid layer superposed a porous layer when the system is heated from below has been considered, firstly, by Sun [1] who showed

بسم الله الرحمن الرحيم

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